ELEMENTS
OF
Plain and Spherical
Trigonometry;

1485.aa

Together with the
Principles of *Spherick*
GEOMETRY,
And the ſeveral
PROJECTIONS of the SPHERE
In *Plano*.

The whole Demonſtrated and Illuſtrated
with Uſeful *Caſes* and *Examples*.

By J. HARRIS, D. D. F. R. S.

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BRITISH MUSEUM
31 OCT 56

TO THE

READER.

THO' there are very many Books on this Subject already extant, ſome of which are very well done; yet I perſwade my ſelf, that the Plainneſs and the Brevity of this Tract will make it very uſeful and ſerviceable to many Perſons to whom ſome larger Books may ſeem both difficult and Tedious: And I am apt to think that a few Hints may be met with here, which will ſerve to give the Reader a much clearer and diſtincter Notion of the Nature of Trigonometry, *than is ordinarily to be met with in Books of this Subject; eſpecially as to the* Rationale *of* Stating a Caſe, *and* Forming a Canon *to work; by which till a Perſon can readily do, he is by no means a Maſter of this Science.*

For tho' a Man may only by Rules contrived to help the Memory, retain a thing a great while, eſpecially if Practice make it ready and familiar to him; yet 'tis, I think, much better, in order to prevent Miſtakes or Blunders; or to avoid being puzzled with ſome Difficulties, which either the Nature of a Queſtion, or the Manner of propoſing it, may ſometimes Occaſion: *'Tis much* *better, I ſay, That the Learner be let into the whole Myſtery, ſee the thing to the very bottom, and be permitted to take no Steps, but where he hath ſolid Demonſtration to ſtand or proceed upon; and then he will be out of danger of either being deceived himſelf, or expoſing the Reputation of his Teacher.*

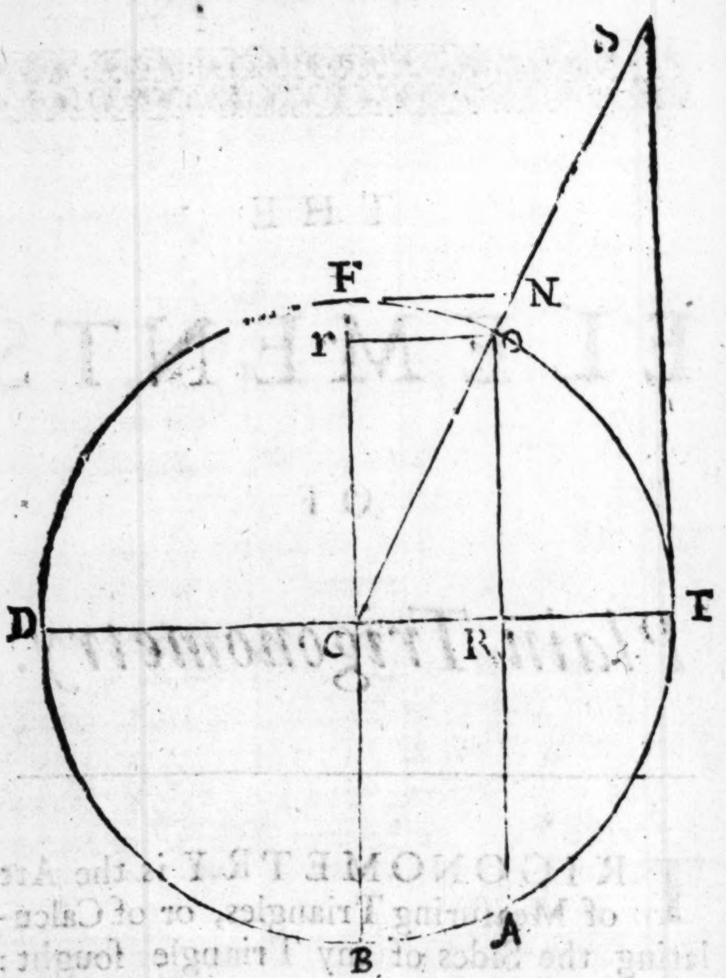
I queſtion not but this little Book will be an Inſtance of the Poſſibility and Falicity of proceeding in ſuch a Method; and will convince any unprejudic'd Perſon, that the Theory *and* Practice *together, of any Part of* Mathematicks, *may be taught as ſoon as the* Practice *alone without the* Theory.

The Reaſon of its Publication was, that it might be in the Hands of my Auditory at the Publick Mathematick Lecture in Burchin-Lane, *when I am explaining this Subject to them; and where, as ſoon as I have finiſhed* Plain Trigonometry, *I ſhall proceed to read* Shperick Geometry, *and* Projection of the Sphere in Plano, *and then* Spherick Trigonometry; *which laſt Science cannot be well underſtood, without the Learner go in the Order and Method I have now mentioned. And this Hint may ſerve to ſhew the great Defect of ſuch Treatiſes of the whole Art of* Trigonometry, *as have nothing of* Spherical Projection, *Introductory to the Doctrine of* Spherick Triangles.

THE

THE ELEMENTS OF *Plain Trigonometry.*

TRIGONOMETRY is the Art of Meaſuring Triangles, or of Calculating the Sides of any Triangle ſought: and it much depends on the Knowledge of the following Lines in and about a Circle.



DEFINITIONS.

1. The Line D T in a Circle is called the Diameter, one half of which, CF, CO, or C T, is called the Radius; and by ſome the whole Sine, becauſe all other Sines are taken out of it.

2. Any right Line, as O A, joining the two Extremities of an Ark, is called the Chord, or Subtenſe, either of the Ark O T A, or of it's Complement to a whole Circle O D A.

3. The Line O R which is perpendicular to the Radius C T, or which is juſt half the Chord O A, is called the right Sine, or moſt uſually the Sine of the Ark O T, or of the Ark O D, ſo that a right Sine is half the Chord of double the Ark.

4. The Difference of any Ark from a Quadrant, be it more or leſs, is called its Complement. Thus O T is the Complement of the Ark O F, being what it wants of being a Quadrant; and 'tis alſo the Exceſs by which the Ark O D exceeds a Quadrant; the Line *r o* is the right Sine of the Complement, and therefore 'tis called the Co-Sine or Sine-Complement of the Ark O T; as O R is the Co-Sine of the Ark F O.

5. A Line, as T S, touching the Circle in the Point T, and perpendicular to the Radius C T, is called a Tangent: and if a right Line be drawn through O from the Centre of the Circle C, meeting with the Tangent in S, that Line S C is called a Secant, and they both limit one another, ſo as to be the Tangent and Secant of the Ark

O T : F N is the Co-Tangent, and C N the Co-Secant of the ſame Ark ; but they are the Tangent and Secant of the Complimental Ark O F.

6. The Line R T intercepted between the Right Sine and the Tangent is called the Verſed Sine, and by ſome *Sagitta.*

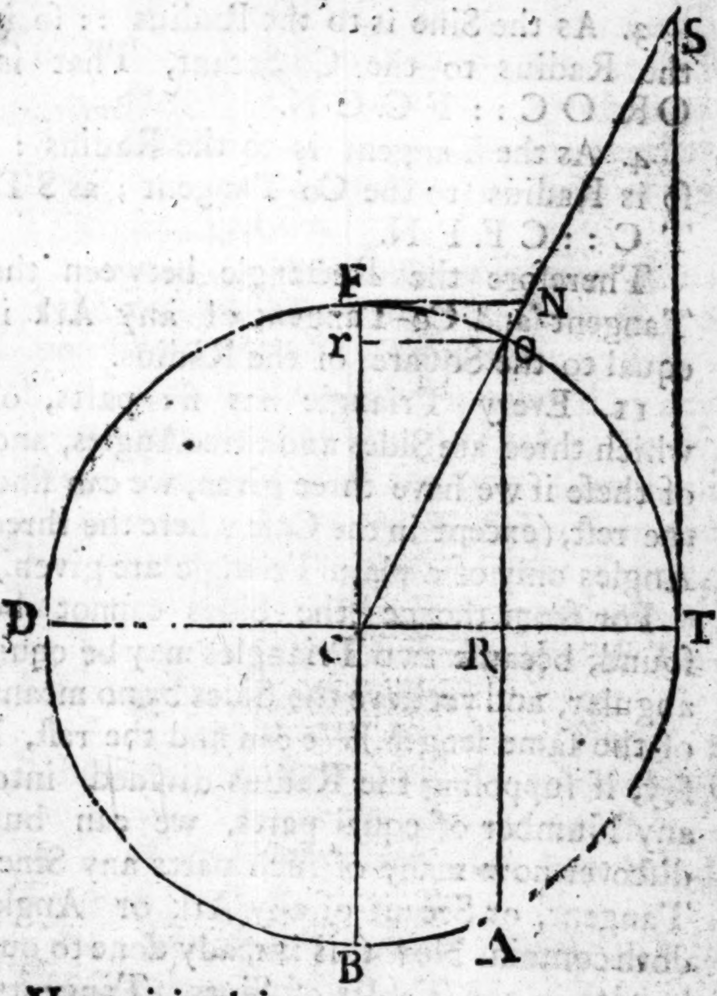
7. Whatever Number of degrees an Ark wants of a Semi-Circle, is called its Supplement.

8. The Line C R, which is the part of the Radius C T lying between the Centre and the right Sine, is always equal to, and may be taken for the Co-Sine *r* O ; and C *r* is equal to the right Sine O R.

9. If any Ark be leſs than a Quadrant, as T O, the Difference between the Radius and the Co-Sine is the Verſed Sine R T ; but if it exceed a Quadrant, as the Ark D F O doth, then the Sum of the Radius and Co-Sine is the Verſed Sine : Thus $DC + CR = DR$ the Verſed Sine of the Ark D F O.

10. The Radius with the Sine and Co-Sine of any Ark (as T O) do make a Right angle Triangle, as O C R, which is ſimilar to the Triangle C S T, made by the Radius, the Tangent, and the Secant. Alſo the Radius, the Co-Tangent, and the Co-Secant make another Triangle ſimilar to the two former

Hence



Hence 'tis plain,

1. That as the Co-Sine is to the Sine, ſo is the Radius to the Tangent. That is, C R. R O : : C T. T S.

2. As Radius is to the Sine : : ſo is the Secant to the Tangent. That is C O. O R : : C S. S T.

3. As the Sine is to the Radius : : ſo is the Radius to the Co-Secant, That is, OR. O C : : F C. C N.

4. As the Tangent is to the Radius : : ſo is Radius to the Co-Tangent ; as S T. T C : : C F. F N.

Therefore the Rectangle between the Tangent and Co-Tangent of any Ark is equal to the Square of the Radius.

11. Every Triangle has ſix parts, of which three are Sides and three Angles, and of theſe if we have three given, we can find the reſt, (except in the Caſe where the three Angles only of a plain Triangle are given.) For from thence the Sides cannot be found, becauſe two Triangles may be equiangular, and yet have the Sides by no means of the ſame length.) We can find the reſt, I ſay, if ſuppoſing the Radius divided into any Number of equal parts, we can but diſcover how many of ſuch parts any Sine, Tangent, or Secant of any Ark or Angle doth contain. Now this is ready done to our hands, in the Tables of Sines, Tangents, and Secants, which we have, with prodigious Induſtry, in Books ready calculated for this purpoſe.

12. When therefore any Triangle is given to be reſolved, the firſt thing we have to do is to conſider, that there is in the

Table of Logarithms, Sines, Tangents, and Secants, a Triangle exactly ſimilar and equal to that which we are required to ſolve, and whoſe Sides are to one another in the very ſame proportion of thoſe of the Triangles propoſed.

2. We muſt underſtand whatever *Ratio* one ſide of the Triangle given hath to the other ſide about the ſame Angle, conſidered as Lengths eſtimated or number'd by any known meaſure, as ſuppoſe Inches, Yards, Miles, Leagues, *&c.* the very ſame hath the two Sides about the ſame Angle in the Triangles in the Tables or in the Tabular Parts; which two things well underſtood, do lead us into the whole Myſtery of Trigonometrical Calculations.

13. Trigonometry is either Plain or Spherical, and both may be reſolved by the means of four Proportions which becauſe of their excellent Uſe are called Axioms.

14. The firſt of which relates to Rectangle plain Triangles; and is this.

AXIOM I.

In a Right-angle Triangle, if either of the Legs be ſuppoſed to be the Radius of a Circle, the other Leg will be the Tangent of the oppoſite Angle, or of the Angle at the Centre ; and the Hypothenuſe will be the Secant of that Angle : But if you imagine the Hypothenuſe to be the Radius of a Circle, then each Leg will be the Sine of its oppoſite Angle, or of the Angles at the Centre ; as is plain from the adjoyning Figures.

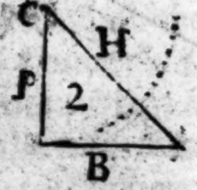
In the firſt of which, B (the Baſe) being made the Radius, P (the perpendicular) is the Tangent of the Angle at the Centre of the Circle *a* which is oppoſite to P and the Hypothenuſe, is the Secant of the ſame Angle.

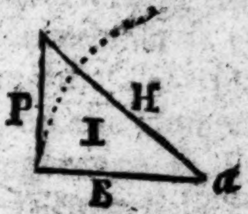
In the ſecond Figure, where P is made the Radius, B is the Tangent of the oppoſite Angle at the Centre *c*.

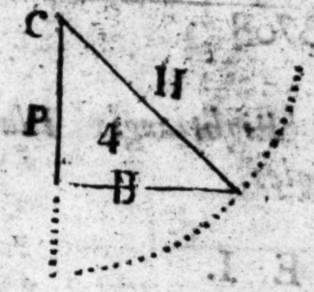
In the third Figure, where H the Hypothenuſe is made the Radius, P is the right ſine of the oppoſite Angle at the Centre. And,

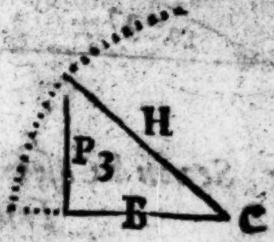
In the fourth Figure, H being alſo made Radius, but C the Centre of the Circle, B will be the Sine of its oppoſite Angle *c*.

The





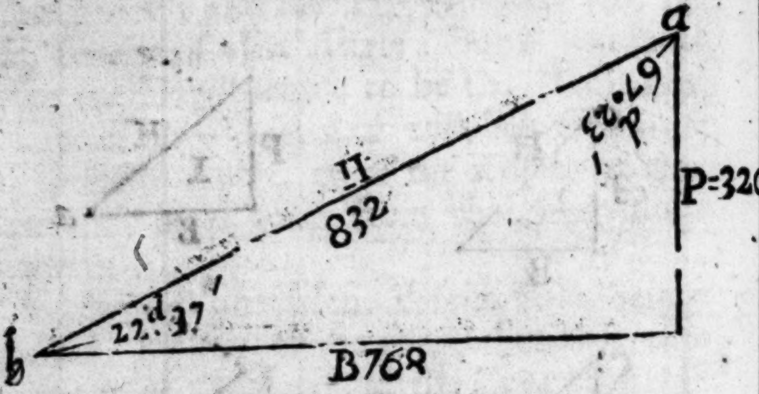




The Seven Caſes of Plain Triangles.

	Given	re-quired	Proportions	Given	re-quired
1	B. *a*, *b*	P	R. B : : Tan. *b*. P	*c*. √ √	*c*
2	B, *a*, *b*	H	S, *a*, B : : R. H	*c*. √ √	*b*
3	B. H	√ √	H. R : : B. S, *a*	*c* √, *b*	√ √
4	B, H	P	H. R : : B. S, *a* ThenR.T *b*:: B.P Or R. B : : T. *b*. P	which finds the √ *a*,	
5	B. P	√ √	B. R : : P. T. *b*	*c* √.	√ √
6	B. P	H	P. R : : P. T. *b* ThenS *b*.P : : R.H	by the laſt *c* √.	*b*
7	H √ √	B	R. H : : S, *a*. B	*b* √ √	*c* √

The



The Seven Cases of Right-angle Plain Triangles.

CASE I.

Given B. *a.* and *b.* Required P ?

Canon $\boxed{R.\ B :: T.\ b.\ P\ ?}$

R = 10.

B = 2.8853612 = 768

T, *b* = 9.6197205 = 22°. 37′.

P = 2.5050817 = 320 = P ?

The General Rule for all Operations in Trigonometry, is, to write down the Numbers found in the Tables according to the Order of the Canon: and then adding together the 2d and 3d Nnmbers, from their Sum ſubſtract the 1ſt, the Remainder is the Logarithm of the Term ſought.

By Gunter's *Line*: Extend the Compaſſes from 45 deg. on the Tangents to 22 deg. 37 min. the ſame Extent will reach in the Line of Numbers from 768 backward to 320.

CASE II.

Given B, *a*, *b*. Req. H?

$$\boxed{S, a . B :: R . H.}$$

$$S, a = 9.9652480 = 67^{\circ}.\ 23'.$$
$$B = 2.8853612 = 768.$$
$$R = 10.$$
$$H = 2.9201132 = 832 = H.$$

By Gunter's *Scale*: Extend the Compaſſes from 67 deg. 23 min. to 90 deg. on the Line of Sines; the ſame Extent will reach from 768 to 832 on the Line of numbers.

The general Method to ſtate any Caſe, or to form the Canon.

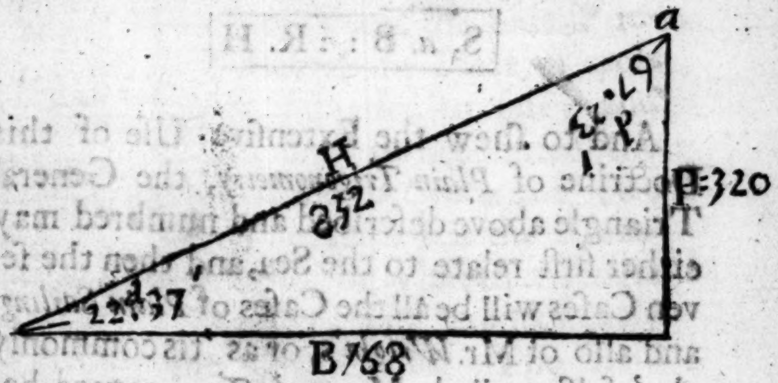
Conſider that the thing ſought muſt always ſtand in the 4th or laſt Place: and therefore *in Caſ.* 1. ſince P, at length is ſought, that muſt be the laſt of the Four Terms; place it therefore laſt with an Interrogation point after it, to ſhew that it is required or ſought.

In the Golden Rule the Second and fourth Terms being always of the ſame nature or kind, and P being a length ſought, and B the only length given; B is neceſſarily determined to be in the ſecond place write it down therefore in that place with four Points after it thus : : to ſhew that the Proportion disjoyns or brakes off there.

3. Conſider that the Hypothenuſe not being either given or ſought, the firſt Axiom determines you to work by Tangents: and the Side given B being ſuppoſed Radius, the Proportion muſt be; As B conſidered, as Radius is to its ſelf conſidered as a Length given : : So will P conſidered as the Tangent of the Angle *b*, be to its ſelf conſidered as a Length ſought. That is,

$$\boxed{\text{R. B : : T, } b\text{. P?}}$$

If



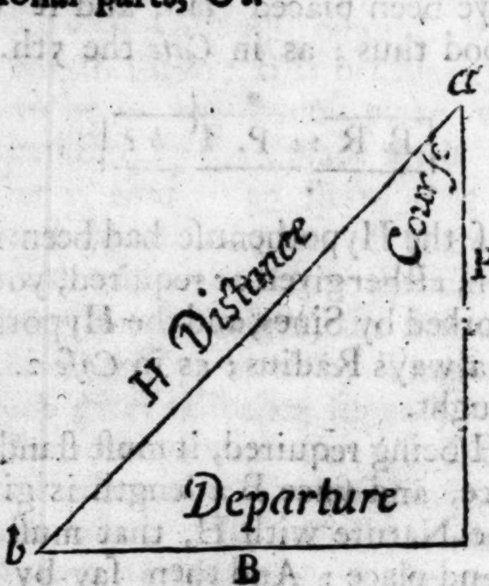
If the Angle *b* had been ſought that muſt have been placed laſt, and it would have ſtood thus; as in *Caſe* the 5th.

| B. R : : P. T, *b* ? |

But if the Hypothenuſe had been in the Queſtion, either given or required, you muſt have worked by Sines, and the Hypothenuſe will be always Radius; as in *Caſe* 2. where H is ſought.

For H being required, it muſt ſtand in the laſt place, and ſince B a length is given of the ſame Nature with H, that muſt be in the ſecond place: And then ſay by *Axiom* 1. As B conſidered as the Sine of the given Angle *a*, is to its ſelf conſidered as a length; ſo is H, conſidered as Radius, to its ſelf as a length ſought. That is, S, *a*. B : : R. H

And to ſhew the Extenſive Uſe of this Doctrine of *Plain Trigonometry*, the General Triangle above deſcribed and numbred may either firſt relate to the Sea, and then the ſeven Caſes will be all the Caſes of *Plain Sailing*; and alſo of Mr. *Wright*'s, or as 'tis commonly tho' falſly called, *Mercator's Too*; regard being firſt had to the way of working by Meridional parts, &c.

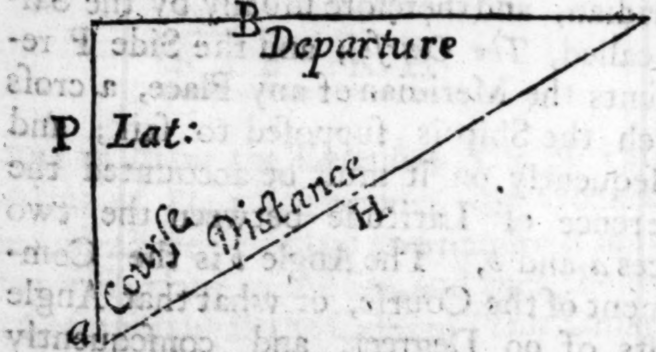


For in this Triangle the Angle *a*, is the Angle of the Rhumb or the Angle which the Line of the Ships Courſe makes with the

Meridian, and therefore uſually by the Sailers called, *The Courſe* ; and the Side P repreſents the Meridian of any Place, a croſs which the Ship is ſuppoſed to ſail ; and conſequently on it muſt be accounted the difference of Latitude between the two Places *a* and *b*, The Angle *b* is the Complement of the Courſe, or what that Angle wants of 90 Degrees, and conſequently known when the Courſe is ſo. The Baſe repreſents the Difference of Longitude, or the Weſting of the Ship in compariſon of the Place at *a*. that ſhe is ſuppoſed to have parted from : and the Hypothenuſe H, repeſents the diſtance Sail'd, or *Run* (as the Seamen call it) or how many Leagues or Miles the Ship hath ſailed from the Place *a* to the Place *b*.

This being underſtood, if this firſt Caſe be made a Caſe of *Plain Sailing*, There will be given Courſe and Departure ; required difference of Latitude and the Canon is.

As Radius is to the Departure in Miles :: So is the Co-tangent of the Courſe, to the Difference of Latitude in Miles.



N. B. Here the Ship being at *a*, is ſuppoſed to Sail to the South Weſtward (or to ſpake exactly, W. S. W.) and therefore her difference of Latitude is reckoned to the South, and her departure to the Weſt: But by Inverting the ſame Triangle, you may ſuppoſe the difference of Latitude North, and the Departure Eaſt; for the North is accounted to lie right before you, and conſequently the Eaſt to the Right-hand, and the Weſt to the Left; wherefore the Courſe now is E. N. E.

2 If you would apply the Doctrine of *Trigonometry*, to the Calculation of Heights, Depths, Diſtances, the ſame Triangle and Numbers will do; regard being had to the Nature of the Terms required and ſought.

For the Perpendicular P, will repreſent any Altitude; and B, will repreſent a Diſtance from the Foot of it meaſured on the

Ground: The Angle at *b* is found by the Quadrant or ſome ſuch Inſtrument; and conſequently you may find P by *Caſe* 1. For,

As Radius is to the Diſtance, from the Foot of the Object : : So is the Tangent of the Angle of Altitude, to the Height of the Place, *i. e.*

R. B : : T, *b*. P.

Or, ſuppoſe the Diſtance B were required from above, by taking the Angle *a*, and finding the Length of P, by a String and Plummet. Then will

R. P : : T, *a*. B

This being promiſed as to the general Uſe of *Trigonometry*, let us proceed to

CASE III.

Given B, H. Req. the Angles *a*, *b*.

H. R : : B. S, *a*

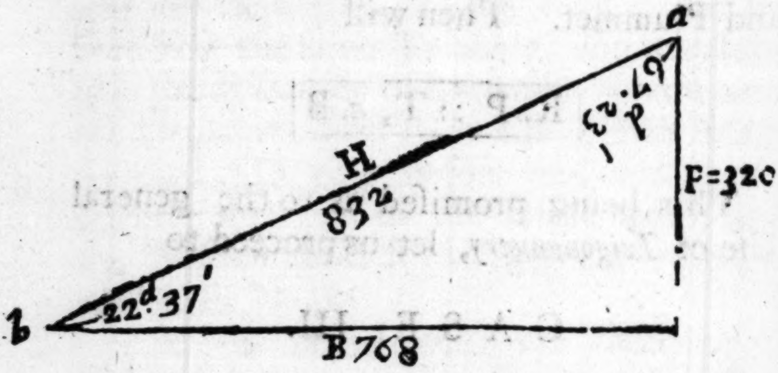
$$H = 2.9201233 = 832.$$

$$R = 10.0000000$$
$$B = 2.8853612 = 768.$$

$$S, a = 9\ 9652379 = 67^\circ.23'.$$
$$= \text{Angle } a.$$

Which Subtracted from 90°, gives the Angle *b* = 22°. 37′.

By *Gunter's* Scale, the Extent from 832, back to 768, in the Line of Numbers, will reach from 90°. to 67°. 30′. in the Line of the Sines.



CASE IV.

Given B, H, Req. P.

Having found *a*, by the foregoing Case, this will be the Theorem.

$$\boxed{R.B : : T.\,b.\,P.}$$

Or,

R. T, *b* :: B P

R = 10.

T, *b*, = 9.6197205 = 22°. 37′.
B = 2.8853912 = 768

2.5050817 = 320 = P.

By *Gunter's* Scale.

The Extent from 45°. in the Tangent-line, back to 22°. 37′. will reach in the Line of Numbers from 768, back to 320.

CASE V.

Given B. P. Req. √√.

B. R :: P. T, *b*

B = 2.8853612 = 768

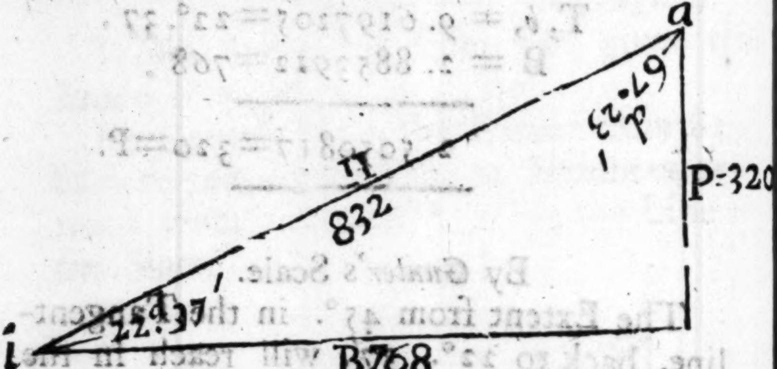
R = 10.
P = 2.5051500 = 320

T, *b* = 9.6197888 = 22°. 37′ = *b*.

Which 22°. 37′ Subtracted from 90°. leaves $a = 67°. 23′$.

By *Gunter's* Scale.

The Extent from 768 back to 320, in the Line, will reach from 45 back to 22°. 37′. in the Tangents.



CASE VI.

Given B, P, Required H.

Having found *b* by the foregoing Case, this will be the Theorem.

$$\boxed{S, b . P :: R . H .}$$

$$S, b = 9.5849685 = 22° 37'$$

$$P = 2.5051500 = 320$$
$$R = 10.$$

$$H = 2,9201815 = 832 = H.$$

By *Gunter's* Scale.

The Extent from 22° 37′ forwards, to 90° in the Line of Lines, will reach from 320, to 832, in the Line of Numbers.

CASE VII.

Given H and √√, Required B?

$$\boxed{\overline{R.\,H}::\overline{S,\,a.\,B}}$$

R, = 10,

2, 9201233 = 832

S, *a* = 9, 9652480 = 67° 23′

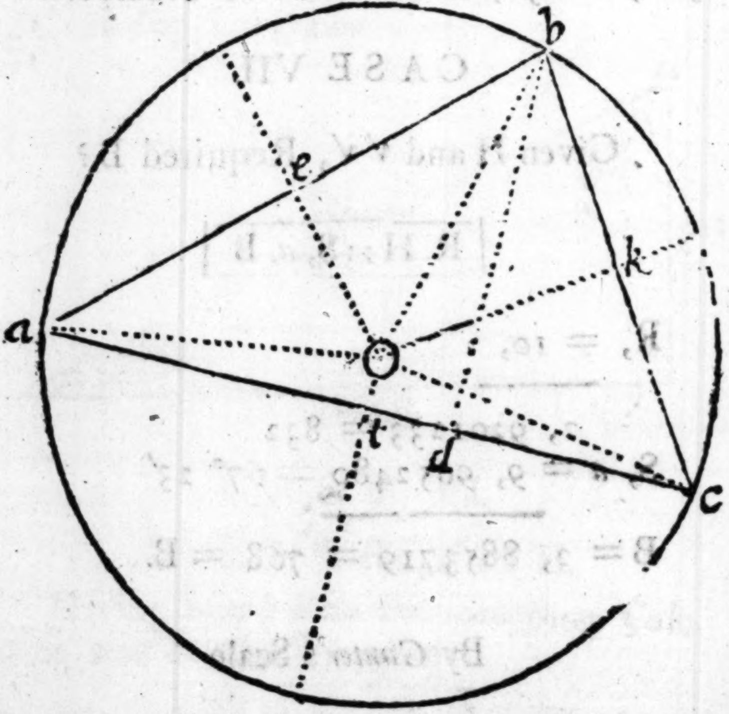
B = 2, 8853719 = 768 = B.

By *Gunter's* Scale

The Extent from 90° in Line of Sines back to 67° 23′ will reach in the Line of Numbers from 832 backwards to 768.

The Reſolution of Oblique Plain Triangles.

Axiom II.



In every Triangle a, b, c, ***the Sides are in proportion to one another, as the Sines of their oppoſite Angles.***

Let fall from the Angle *b*, a Perpendicular to the Baſe *a c*: For then the whole will be reſolved into two Right Angle Triangles;

and conſequently by Axiom the firſt $ab. R :: bd. S, a.$ alſo $bc. R :: bd. S, c.$ wherefore $ab. bc :: S c. S, a.$ by Equality of Proportion.

Otherwiſe thus ; Draw a Circle about the Triangle from whoſe Centre O, let the Perpendicular $O e$, $O k$ and $O t$, be let fall to the three Sides of the Triangle, and the Lines $o a$, $o b$, and $o c$ be drawn to the three Angles.

The Sides of the Triangle will be biſected by the Perpendiculars, and conſequently ae will be $= eb$, $bk = kc$ and $ct = ta$. wherefore as the whole Line ab is to bc : : ſo will the half ſide ae be to the half kc ; but ae and kc, are the Sines of the Angles at the Centre aoe and koc, which Angles at the Centre are ſeverally equal to the Angles of the Triangle c and a, becauſe they ſtand on half the Arks, that the Angles of the Triangle do ; wherefore $ab. bc :: S, c. S, a.$

Let H, B and O, be the Sides of an Oblique Plain Triangle a, b and c, its three Angles ; here becauſe there is no Right Angle, three things muſt be always given ; that is, either two Sides and one Angle, one Side ard two Angles, or elſe all the three Sides.

From whence will arrive the three former of the ſix following Caſes.

	Given	Required.	Proportions.
1	H. O. *c*.	*b*.	O, S *c* : : H. S, *b*. Note that the Angle *b* is ambiguous, and you muſt collect from the Circumſtances of the Triangle, whether it be Obtuſe or Accute.
2	H. O. *c*.	B.	Here firſt find the Angle *b* by *Caſe* 1, thence *a* will be known by taking the Sum of $b + c$. from 180°. Then S, *c*. H : : S, *a* Or S, *b*. O : : S, *a*. B.
3	*c*. *b*, O,	H.	S, *c*. O : : S, *b*. H.
4	H. *a*. O.	*c*, *b*.	$H + O$. $H - O$: : T $\frac{1}{2}$ Z of the oppoſite Angles to T $\frac{1}{2}$ X of the oppoſite Angles, and $\frac{1}{2}$ Z $+$ $\frac{1}{2}$ X $= b$. and $\frac{1}{2}$ Z $-$ $\frac{1}{2}$ X $= c$
5	H. *a*, O.	B.	Find the Angles by the former Caſe, and then S, *b*. H : : S, *a*. B. or S, *c*. O : : S, *a*. B.
6	H. O. B. 3. Sides	3 Angles.	B. $H + O$: : $H - O$. X. and then $\frac{1}{2}$ B $+$ $\frac{1}{2}$ X $=$ C B and $\frac{1}{2}$ B $-$ $\frac{1}{2}$ X $=$ B *b*. Then will H. R : : C B. Co-ſine Angle C. and O. R : : *b* B, Co-ſine Angle *b*. Axiom 1

And the three laſt are ſolved by the help of the two following Axioms.

Axi-

AXIOM III.

As the Sum of the Legs about the Angle given, is to their Difference :: So is the Tangent of half the Sum of the other two Angles, to the Tangent of half their Difference.

Now the Sum of the other two Angles is known, being what the given Angle wants of 180 Degrees and their Difference is now found; add therefore their half Sum, and half Difference together, and it gives you the *greater of the two Angles ſought*; and half the Difference Subtracted from the half Sum leaves the leſſer Angle ſought. And thus having found the Angles; if the ſide oppoſite to the former given Angle be ſought, it will be found eaſily, by *Pardie's Axiom*, That the Sides are as the Sines of the Angles.

The Demonſtration of the Third Axiom is briefly thus,

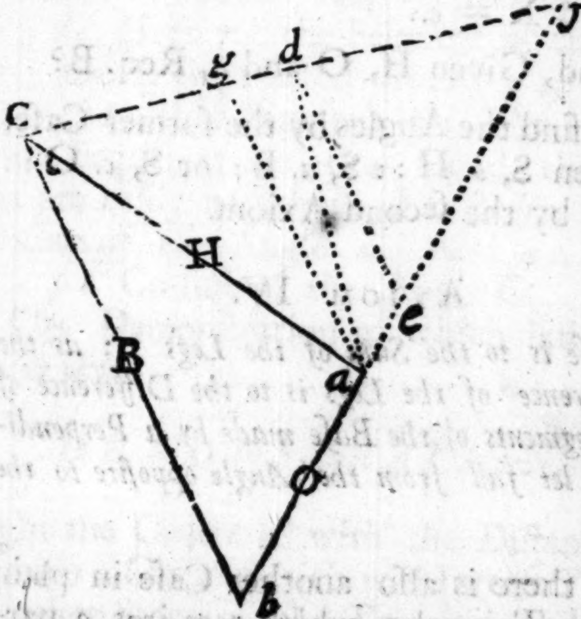
DEMONSTRATION.

I ſay the Sum of the Legs of any Angle *a*, is to their Difference :: as the Tangent of ½ the Sum of their oppoſite Angles, is to the Tangent of half their Difference.

Produce O one of the given Legs of the Angle given, till af become equal to H or C a, and then biſect bf in e join cf, and biſect it alſo in d: Draw ad, which will be Perpendicular to cf (2. 16.) and draw de, which will be Parallel to cb: (6. 92.) Then will the Angle $cad + daf$: i e. to the half of caf. which External Angle $caf = c + b$: That is, to the Sum of the oppſite Angles required.

Drawthen ga, Parallel to cb; ſo will the Angle gac be equal to the alternate one c, And if from $\frac{1}{2}$ the Sum of the oppoſite Angles you take the leſſer Angle: *i. e.* if from cad, you take gac, there will remain the Angle gad equal to half the Difference of the oppoſite Angles.

And ſo alſo, if from be half the Sum of the Legs you take O the leſſer Leg, there will remain ae equal to half the Difference of the Legs. And then ſince the Triangle cad is Right-angled, (by what is proved below in the next Figure ſave one) if ad be made Radius, cd will be the Tangent of the Angle cad (i.e. the Tangent of $\frac{1}{2}$ Sum of oppoſite Angles;) and in the little Triangle gad, gd will be Tangent of the Angle gad (i. e. the Tangent of $\frac{1}{2}$ Difference of oppoſite Angles.) But the Segments of the Legs of any Triangle cut by Lines parallel to the Baſe being Proportional *e. b. e a i : e d. d. g* : That is in



words, *Half the Sum of the Legs is to half their Difference : : as the Tangent of* $\frac{1}{2}$ *the Sum of the oppoſite Angles is to the Tangent of* $\frac{1}{2}$ their Difference, but Whole are as their Halves : Wherefore the Sum of the Legs is to their Difference : : As the Tangent of $\frac{1}{2}$ the Sum of the oppoſite Angles, is to the Tangent of half their Difference. Q. E. D.

Whence the two following Caſes will eaſily be ſolved.

Firſt, Given H, O and *a* Req *c*, *b*,

For H+ O. H — O : : As T, $\frac{1}{2}$ Z op-

oppoſite Angles, is to T, $\frac{1}{2}$ X oppoſite Angles, and then, $\frac{1}{2} Z + \frac{1}{2} X = b$. and $\frac{1}{2} Z - \frac{1}{2} X = c$.

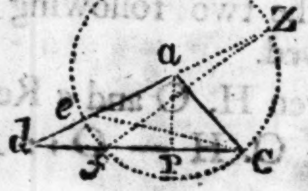
Second, Given H, O and *a*, Req. B?

Firſt find the Angles by the former Caſe, and then S, *b*. H : : S, *a*. B : or S, *c*. O : : S, *a*. B. by the ſecond Axiom.

AXIOM IV.

The Baſe is to the Sum of the Legs : : as the Difference of the Legs is to the Difference of the Segments of the Baſe made by a Perpendicular let fall from the Angle oppoſite to the Baſe.

For, there is alſo another Caſe in plain Oblique Triangles, which requires a particular Axiom to ſolve it; and that is, where all three Sides are given to find the Angles. Here let fall a Perpendicular from any Angle to its oppoſite Side as *a p*, and then ſay, As the Side *dc* is to *d a* + *a c* the



Sum of the other two Sides : : So is the Difference of thoſe two Sides $da - ac$ to a fourth Number. Half of which added to $\frac{1}{2} dc$, gives you the Segment of the Baſe dp; and if Subtracted from $\frac{1}{2} dc$, it will leave the other Segment pc. And when thoſe Segments are thus found, the Angles are eaſily had thus: da. *Radius* : : dp. Co-ſine of the Angle d. And ac. *Radius* : : pc. Co-ſine of the Angle C.

The Demonſtration of which laſt Axiom is this,

DEMONSTRATION.

On the Centre a, with the Diſtance ac deſcribe a Circle, which will Interſect both the other Sides of the Triangle, and then zd will repreſent the Sum of the Legs da and ac : ae will repreſent their Difference, and df will repreſent the Difference of the Segments of the Baſe made by the fall of the Perpendicular aP.

Then I ſay, $dc . dz : : de . df$. That is, *The Baſe is to the Sum of the Legs : : As the Difference between the two Sides is to the Difference of the Segments of the Baſe*; as is apparent from *Prop.* 67. of *Pardie*'s Sixth Book, after drawing the Prickt Lines ec and fz.

And then the Caſe will ſtand thus,

3. Given H, O, B, all the Three Sides, Required the Angles. See the Figures before.

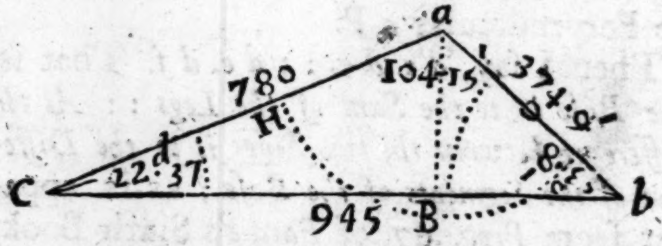
I ſay by this Axiom B. H + O : : H — O. X, which expreſſes the Difference of the Segments of the Baſe, = *d f* in the Figure.

And then having found X $\frac{1}{2}$ B + $\frac{1}{2}$ X = *dp*, the greater Segment, and $\frac{1}{2}$ B — $\frac{1}{2}$ X = *pe* the leſſer, by which means the two Right-angled Triangles *a d p* and *a p c* will be ſolved eaſily : For *d a. Radius* : : *d p.* Co-ſine of the Angle *d* and *a c. Radius* : : as *p c.* to the Co-ſine of the Angle *e*.

The Operation of the Six Caſes of Oblique angled Triangles.

CASE I.

Given H, O, *c*, Required *b*.



O. S, *c* : : H. S, *b*.

The Numbers of the Triangle are as follows.

√ c = 22°. 37′
√ b = 53. 08
√ a = 104. 15 Or, its Suppl. = 75°. 45′

H = 780
O = 374. 9
B = 945

O = 2. 5739154 = 347. 9

S, c = 9. 5849685 = 22°. 37
H = 2. 8920946 = 780

Sum = 12.4770631

S, b = 9. 9031477 = 53°. 8′

CASE II.

Given H, O, c. Required B.

$$\boxed{\text{S, } c\text{. O : : S, } a\text{. B.}}$$

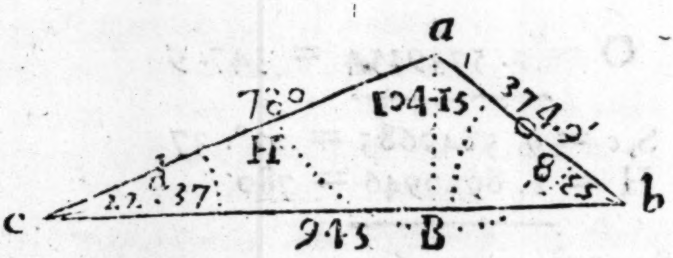
S, c = 9. 5849685 = 22°. 37′

O = 2.5739154 = 374.9
S, *a* = 9.9864273 = 104°.15′

Sum = 12.5603427

B = 2.9753742 = 945

CASE III.



Given *c*, *b*, O. Req. H?

S, *c* = 9.5849685 = 22°. 37′

O = 2.5739154 = 374.9
S, *b* = 9.9031084 = 53°. 8′

Sum = 12.4770238

H = 2.8920553 = 780

CASE

CASE IV.

Given H, *a*, O. Required *b*, *c*.

$$\boxed{H+O . H-O :: T,\tfrac{1}{2}Z\,op.\surd\surd\ T,\tfrac{1}{2}X\,op.\surd\surd}$$

$H+O = 1154.\ 9$
$H-O = 405.\ 1$
$\tfrac{1}{2}Z\,opp.\angle\angle = 37^\circ.\ 52'$

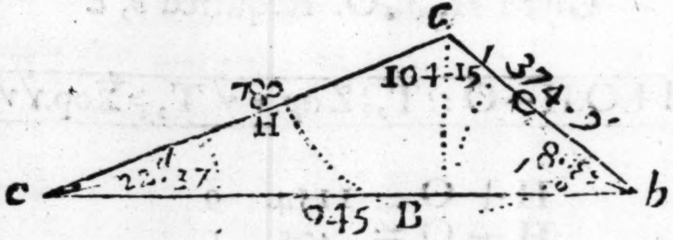
$H+O = 3.0625820 = 1154\ 9$

$H-O = 2.6074550 = 405.\ 1$
$T\ \tfrac{1}{2}Z = 9.8907254 = 37^\circ.52'$

$Sum = 12.4981804$

$T,\ \tfrac{1}{2}X.\ 9.4355984 = 15^\circ.15'$
Then will $\tfrac{1}{2}Z + \tfrac{1}{2}X = 53^\circ\ 7' = b$:
and $\tfrac{1}{2}Z - \tfrac{1}{2}X = 22^\circ\ 37 = c$.

CASE V.



Given H, *a*, O. Required B.

Find the Angles *b* and *c*, by *Caſe* 4. then

S, *b*. H : : S, *a*. B

S, *b* = 9. 9031084 = 53°. 8

H = 2. 8920946 = 780

S, *a* = 9. 9864273 = 104°. 15′ *

Sum = 12.8785219

B = 2. 9754135 = 945

* This is the Sine of 75. 45, the Supplement of 104. 15, to a Semicircle.

CASE

CASE VI.

Given H, O, B. Req. Three Angles.

$B . H+O : : H-O . X.$

$\frac{1}{2}B+\frac{1}{2}X=CB$ & $\frac{1}{2}B-\frac{1}{2}X=Bb$

$H . R : : Bc . \Sigma c :$ & $O . R : : Bb . \Sigma b$

$B = 2.9754318 = 945$

$H+O = 3.0622058 = 1154.9$

$H-O = 2.6074540 = 405.1$

$\frac{1}{2}B = 472\frac{1}{2}$ $\quad 5.6696958 =$

$\frac{1}{2}X = 247\frac{1}{2}$

$2.6942280 = 495$

$H = 2.8920553 = 780'$

$\frac{1}{2}B+\frac{1}{2}X=720$

$\frac{1}{2}B-\frac{1}{2}X=225$ $\quad R = 10.$

$BC = 2.8573225 = 720$

$\Sigma c = 9.9652672 = 22^{\circ}\,37'$

$$O = 2.5739154$$

$$R = 10.$$
$$Bb = 2.3521825$$

$$\Sigma b = 9.7782671 = 53°08'$$

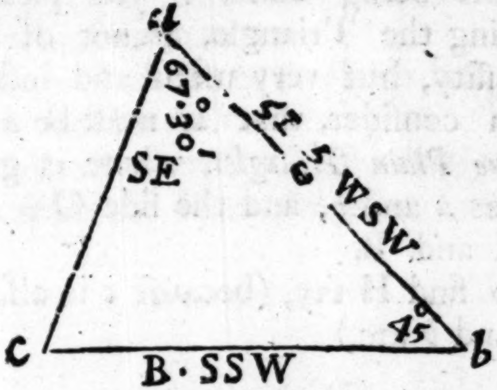
And when thus the Nature, Reaſon, and Method of Operation in the Calculating of the Sides and Angles of oblique Triangles, is fully underſtood; its Application, Uſe, and Practice, will be very eaſie and plain, though its extent be very large and ample. For,

Firſt, If you have a Mind to apply it to what they call *Oblique Sailing*, or the Doctrine of *Oblique Plain Trigonometry*, applied to *Sailing*; your former Triangle and Caſes will reach all that you can deſire.

As ſuppoſe a Ship Coaſting along by the Shoar from the Place *b*, ſets an Head Land by her Compaſs (as *c*) and finds it to bear from her S. S. W. then ſhe Sails on, W. S. W. 51. 5 Miles or Minutes to *a*; and then finds that the Head Land bears from her full S. E.

'Tis required to determine her diſtance from this Head Land when ſhe was at *b*, and now ſhe is at *a*.

First,



First, To Plot the Triangle.

Draw a ſtrait Line as B, repreſenting the firſt bearing of the Head Land, which was S. S. W. Then becauſe the Ship Sail'd Weſt South Weſt, make the Angle *b*, equal to 45°, and ſo will the Line O repreſent the Diſtance ſailed, and *b* will be the Courſe. Prick off the Diſtance run, *viz.* 51. 5 Miles from *b* to *a*. Then becauſe the Ship ſailed W. S. W. the contrary Rumb from *a* to *b*, muſt be E. N. E : And ſince the South Eaſt Rumb makes with that, an Angle of 67° 30', you muſt make the Angle at *a*, juſt 67° 30'. So will the Line H, when drawn, interſect the Line B in the Point of the Situation of the Head Land *c* ; and by that means will the Triangle be compleated.

This being done, if you pleaſe, (tho' plotting the Triangle, is not of abſolute Neceſſity, but very uſeful and inſtructive.) Then conſider, that it muſt be a Caſe of *Oblique Plain Triangles*, where is given the Angles *a* and *b*, and the ſide O. Required H and B.

To find H ſay, (becauſe *c* is alſo known if *a* and *b* are.)

$\boxed{S,c.\ O :: S,b.\ H}$

Operation.

S, *c* = 9. 9656153 = 67° 30′

O = 1. 7118072 = 51. 5 Miles.
S, *b* = 9. 8494850 = 45°

11. 5612922

H = 1. 5956769 = 39. 4 Miles.

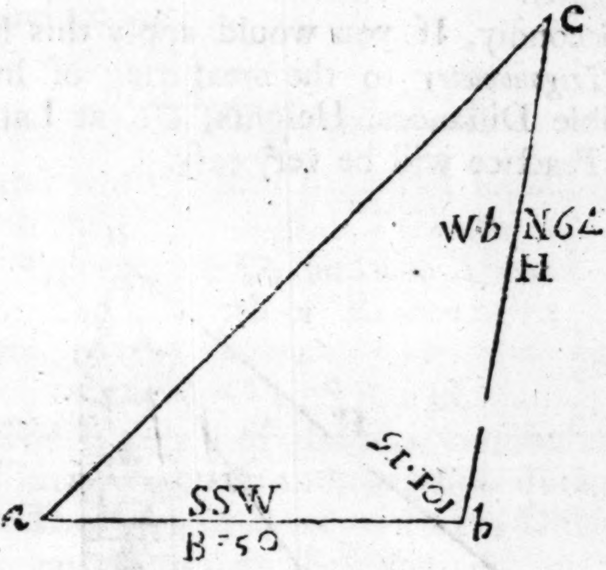
And to find B, you may ſay,
As S, *b*. H : : S, *a*. to B :

Or,

As S, *c*. O : : S, *a*. to B.

I ſhall give but one Inſtance more in this Matter, which is this.

Suppoſe a Ship ſail S. S. W. 50 Leagues, and then W. by N. 64 Leagues: What was her direct Courſe, and what is her Diſtance from the Place ſhe went from?



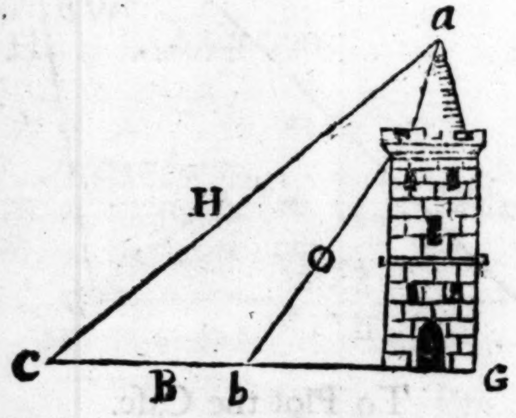
To Plot the Caſe.

Let *b* be the Place from whence the Ship ſailed, and let B repreſent the S. S. W. Rumb, and the Diſtance thereon run = 50 Leagues, from *b* to *a*.

Then ſince the firſt Courſe was two Points, and the ſecond ſeven Points from the Meridian, make an obtuſe Angle at *a*, equal to 9 Points, or 101° 15′: And on the Leg H, ſet 64 Leagues, then Drawing the Side *ac*, the Triangle will be formed.

In which you have two Legs, H and B forming the Angle *a*, and that Angle given. And this is *Caſe* 4. of *Plain Oblique Triangles.*

Secondly, If you would apply this Part of *Trigonometry* to the meaſuring of Inacceſſible Diſtanccs, Heights, *&c.* at Land; the Practice will be very eaſy.



Suppoſe a Tower, Steeple, *&c.* as *a* G, whoſe Height you would take, but can meaſure no nearer then from *c* to *b*, but know the Length B *b*, is = 100 Yards.

Here you can take the Angle *a b* G, with your Quadrant, and conſequently the obtuſe contiguous One, *c b a*, is known.

The Angle *c*, is known after the ſame way by the Inſtrument: Wherefore having in the obtuſe angled Triangle H O B, the Baſe B, and the Angles *c* and *b*, you may find O by *Caſe* 3. of *Oblique Plain Triangles*: For ſince *c* and *b* are known, their Sum ſubtracted from 180, will leave *c a b* known. Therefore ſay,

As S, *a*. B : : S, *c*. O.

And when is thus known or found, having in the Right-angled Triangle, *b a* G, the Hypothenuſe O, and the Angle *b*.

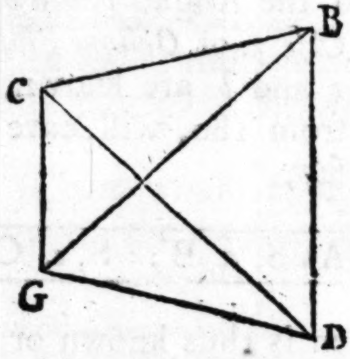
By *Caſe* the 7th of Right-angled Triangles.

As R. O : : S, *b*. *a* G.

The Altitude of the Tower ſought.

Thirdly, Suppoſe an Inacceſſible diſtance, as B D. which imagine to be the Diſtance between two Forts, Baſtions, *&c.* on the Wall, or Line of an Enemy's City, or Camp; and that becauſe of the Cannon, *&c.* you can go no nearer then the Line G C, but can meaſure from G to C, and at each Station take Angles with an Inſtrument.

Having taken then by the Theodelite, *&c.* the Angles G C D, and C G D, and meaſured G C: And alſo having meaſured the Angles B G D, and B C G, *&c.* This premiſed, which is eaſily done by the Inſtrument.



You have in the Triangle C B G, the Side C G, and the two Angles G, and G C B : Wherefore, alſo the Angle C B G. Say, therefore,

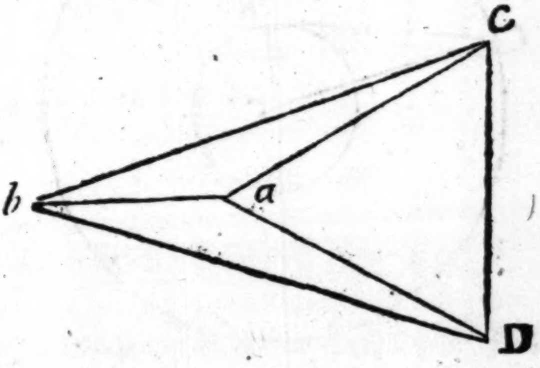
As S, B. is to G C : : S, C. to G B.
And conſequently G B is found.

Again, in the Triangle C D G, the Angles G C D, C G D, and the ſide C G being given, the ſide G D will be found by this Proportion.

As S, D. C G : : S, C. G D.
And conſequently G D alſo is found.

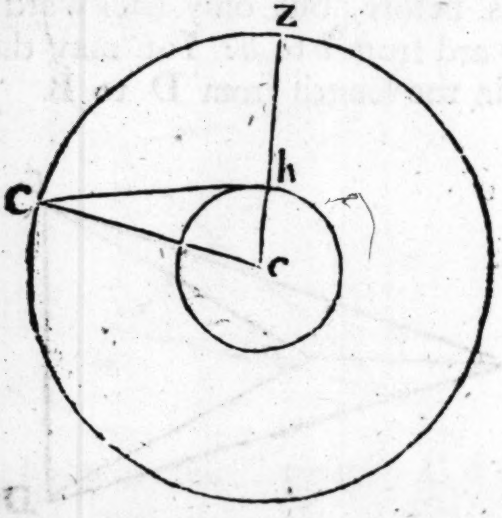
And now having in the Triangle G B D, the two Sides G B and G D, and the Included Angle G ; you can firſt find the remaining Angles ſeverally by *Axiom* 3. and then the Side B D by *Axiom* 2.

Or, If not being able to get nearer than the Point *a*, you could not meaſure ſideways as before, but only backward to *b*, or forward from *b* to *a*: You may then eaſily gain the length from D to B.



For placing the Inſtrument at *a*, you can take the two Angles *b a* D, and *b a* C, and take alſo C *a* D; meaſure then from *a* to *b*, and at *b* take alſo the two Angles *a b* D, and *a b* C. Then can you eaſily gain the the ſides *a* C, and *a* D, in the two Triangles *b a* C, and *b a* D; and having before taken by the Inſtrument the Angle C *a* D, you may find the Side C D in that laſt Triangle *a* C D, by the ſecond and third *Axioms* of *Plain Oblique Trigonometry*. Alſo

Fourthly, On the Application of this Part of *Trigonometry*, to the Doctrine of *Aſtronomy*, depends the Method for finding the Parallax and Diſtance of a Planet, or Star.



Let *c* be the Centre of our Earth, and *h* a Point on its Surface ; at which, an Obſerver at *h*, takes with an Inſtrument, the Angle ☾ *h z*, on the Diſtance of Meridian Moon ☾, from her Zenith at *z*.

Her true Diſtance from the Zenith, is known by the Aſtronomical Tables ; which is the Angle ☾ *c z* : But the obſerved Angle ☾ *h z*, being external to the Triangle, ☾ *h z* will be = to ☾ + *c*.

Where take *c* from it, and the Remainder is the Angle *h* ☾ *c* = to the Moons Parallax ; whoſe Subtenſe is *h c* = to the Earth's Semi-diameter.

Suppoſe that *h c* be 4000 Miles; then in the Triangle ☾ *h c*, there are all the Angles, and the ſide *h c* known.

Wherefore,

As S, ☾. *h c* : : S, *h*. ☾. *c*.
Which is the Moon's Diſtance from the Centre of our Globe.

And alſo, as

S. ☾. *h c* : : S, *c*. ☾ *h*.

The Moon's Diſtance from the Place of Obſervation.

Directions for the Uſe of the Tables of LOGARITHMS, SINES, TANGENTS, *&c.*

THESE Tables are in every ones Hands, and may be bound up with this Treatiſe.

The Table of Logarithms ſerves to afford you Artificial Numbers to be uſed inſtead of thoſe natural Numbers which expreſs the Length of the Sides of all Plain Triangles: that ſo you may work by *Addition* and *Subtraction*, inſtead of *Multiplication* and *Diviſion.*

How theſe Tables are made, is not the Deſign of this ſhort Tract to inform you; but how to uſe them in Calculation, muſt be well underſtood.

Suppoſe then you were to look for the Logarithm of the Side B, in *Caſe* 1. of Right-angled Triangles; whoſe Number is 768.

Look in the Table of Logarithms, under the Word *Number*, or the Letter *N*, in the firſt Column towards the Left-hand, where are all Numbers in their natural Or-

Order, from 1. to 1000: and right againſt 768, you will find 2.8853612. if you uſe *Ulack*'s Tables, which I think are the beſt.

But if you uſe the Logarithms in the common Books of *Trigonometry*, the Logarithm ſought, will be leſs by one Figure, *viz*. It will be 2.885361. Where you muſt take Notice, That the firſt Figure 2. with the Point after it, is what we call the Characteriſtick, or Index of a Logarithm; becauſe it ſhews the Nature of the abſolute correſponding Number, being always an Unite leſs than its Number of Places.

Thus becauſe the Number 768, conſiſts of three Places, the Index of its Logarithm muſt be 2: But if it had been 7680, its Index would have been 3, becauſe the Number hath four *Real Places*; *Real Places*, I ſay, or *Places of Integers*: For if it had been in Part a Decimal Fraction, its Index would have related only to the integer Part; and will be defective or Negative, if the Number be entirely Fractional.

Thus the Index of *768.6* is as before, but 2, and the Index of *76.86* is but 1, of *7.686* is 0, and of *.7686* (according to Sir *Jonas Moore*'s Way, who rightly adviſes, when you have to deal with the Logarithms of Fractions, to take the Complements of the Indexes to 10) the Index of 7686 will be $\overline{9}$.

And thus may the Logarithm of any Number be readily found in the Tables, and a proper Index prefix'd to it, which muſt always be done, whether the Table give you the Index or not; for if it be not there, as it is not in thoſe Tables which are Sir *Jonas Moore's Mathematical Compendium*, and ſome other Books, you muſt always ſupply it by the Rule above given.

2. To find the proper Number anſwering to any Lagarithm given. As ſuppoſe the Logarithm 2.8853713 were given, to know what Number it repreſents; which is the Logarithm found by Operation, to belong to B, in *Caſe* 7. of Right-angled Triangles.

Imagine its Characteriſtick to be 3, (which is the higheſt the Tables extend to) and then look in the Tables for the Logarithm of 3.8853713; or omitting the Characteriſtick, look for 8853713, in the proper Column under the Word Logarithm: You will not find it there exactly; but that which comes neareſt to it, and is yet leſs than it, is 3.8853612, which anſwers to the abſolute Number 7680.

From which Number cut off the Cypher at the End, becauſe the Index of your Logarithm was really but 2. and not 3. and the remaining three Figures 768, are to be taken for the true Number, or Length of the ſide B in the Triangle; but if inſtead of 2. the Index of the Logarithm had been 1. or 0.: Then you muſt have cut off one or two more Places from the Number 7680, and then the Number would have been accordingly, 76.80, or 7.680.

And if the Logarithm be not found there exactly, and that its Index ſuppoſe alſo be 4. find it firſt to four Places, as is before ſhewn.

And Note the common Difference under the Letter D, in the laſt Column of the Logarithms in the Book: Next take the Difference between the Logarithm propoſed, and that which is next leſs than it in the Tables; and then having Recourſe to the Tables of proportional Parts, which are uſually Printed with the Logarithms; Seek there under the Letter D, the before found common Difference under the Column D in the Logarithms; then run your Eye along in an Horizontal Line, till you find the Number neareſt expreſſing the Difference of the Logarithm, which you found by Subtraction, and then at the Top you will have the 5th Figure to be added or annexed to the four others before found. Thus,

Suppoſe the Logarithm had been as above, 2.8853713, but with 4. inſtead of 2. for its Index: That is, Suppoſing it to belong to a Number conſiſting of five Places.

The Number found then was 7680, againſt which, under the Letter D, ſtands 565 for the common Difference, and the Difference between the Logarithm given, *viz.* 4. and the next leſs in the Tables, *viz.* 4. is 101: And here though I can neither find 565, nor 101 in the Tables of Parts proportional exactly; yet taking the neareſt to them, I find at the Top the Figure 2, which is to be annexed to 7680, and then it will be 76802; which would be the right Number anſwering to the Logarithm given, if its Index had been 4.

8853713
8853612
———
101

The

The Uſe of the other Tables of Sines, Tangents, *and* Secants, *is very eaſy and plain.*

For if you would find, ſuppoſe the Logarithmick Sine of 22°. 37′. Look for 20°. on the top of the Page, and 37′. in the firſt Column for Minutes on the Left-hand, and marked with M; and then under the Word Sine, you have *9.5849685* for the Logarithmick Sine, and *9.6197205* for the Tangent, and 10.0247521 for its Secant.

But to look for the Sine, Tangent, or Secant of any Degrees above 45, you muſt in moſt Books of Tables ſeek the Degrees at the bottom of the Right-hand Page; and the odd Minutes muſt be reckon'd upwards alſo in the Column for Minutes on the Righr-hand of that Page.

As if you were to look for the Sine, Tangent, and Secant of *67°. 23′.* You will find them in the ſame Page and Line with thoſe of 22° 37′. (whoſe Complement to *90°.* is *67°. 23′.)* but only you muſt account from the Bottom upwards, as is above ſaid; and the Sine will be 9.965248, the Tangent 10.380279 and the Secant 10.415031.

And as eaſy as is it to find the Degrees and Minutes anſwering to any Logarithmick Sine, Tangent, or Secant, only obſerve when you cannot find it exactly, always to take the next leaſt : Thus,

If you were to ſeek what Degrees and Minutes anſwer to the Logarithmick Sine of *9.9652379*, you will not find it exactly in the Tables, but the next Leſs is *9.9651953*, which belongs to *67°. 22'*.

OF THE *Orthographick Projection* OF THE SPHERE.

PART I.

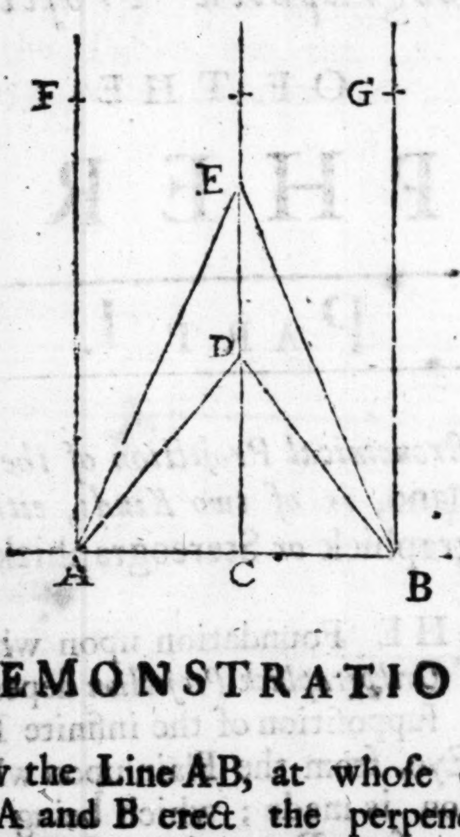
The Aſtronomical Projection of the Sphere *in* Plano, *is of two Kinds, either* Orthographick *or* Stereographick.

THE Foundation upon which the *Orthographick Projection* depends, is a ſuppoſition of the infinite Diſtance of the Eye, from the Plain upon which the Projection is made; which being granted 'twill be eaſy to Demonſtrate this firſt *Prop.*

PROP.

PROP. I.

The Rays by which the Eye at an infinite Diſtance ſees any Object, differ infinitely little from parallel Rays.



DEMONSTRATION.

Draw the Line AB, at whoſe extream points A and B erect the perpendiculars A F and B G: I ſay, the Eye being ſuppoſed at an infinite Diſtance ſhall ſee AB under the parallel Rays A F and G B, nearly:

for if you Biſſect A B, and Erect at the point C the infinite Line CDE, and let the Eye be ſuppoſed firſt in D, a Point nearer the Object, and then in E a point more remote; (By the 21 *Prop. e.* 1. *Eucl.*) the Angle ADB is greater than the Angle AEB, and conſequently the Angles DAB and DBA are leſs than the Angles EAB and EBA; wherefore the Lines EA and EB come nearer to the Perpendiculars AF and BG, than the Lines AD and DB do; and the farther the Eye is removed from the Object, the Angles at the Baſe AB approach ſtill nearer and nearer to two Right ones; for their Complements EAF and EBG ſtill grow leſs and leſs: And if the Eye be removed to an infinite Diſtance, that is, if E be at infinite Diſtance from AB, then will the Angles EAF and EBG be infinitely little, and conſequently the Angles at the Baſe EAB and EBA will differ by an infinitely little Difference from the two right Angles FAB and GBA; That is, EA and EB will differ infinitely little from parallel Lines. Q. E. D.

DEFINITIONS.

I. The *Orthographick Projection* of the *Sphere*, is a trnsferring the Repreſentation of the

Sphere, ſeen at an infinite Diſtance, together with its Circles, into a Plain, by Rays perpendicular to that Plain.

II. That plain into which the Sphere, together with its Circles, is ſo transferred, is called the *Plain* of the *Projection.*

III. A point in the Surface of the Sphere, is there projected into a Point, where a perpendicular Ray paſſing thro' the Point to be projected, meets the Plain of the Projection.

> *N. B.* When I ſpeak of Perpendiculars, I mean Lines or Rays perpendicular to the Plain of the Projection.

PROP. II.

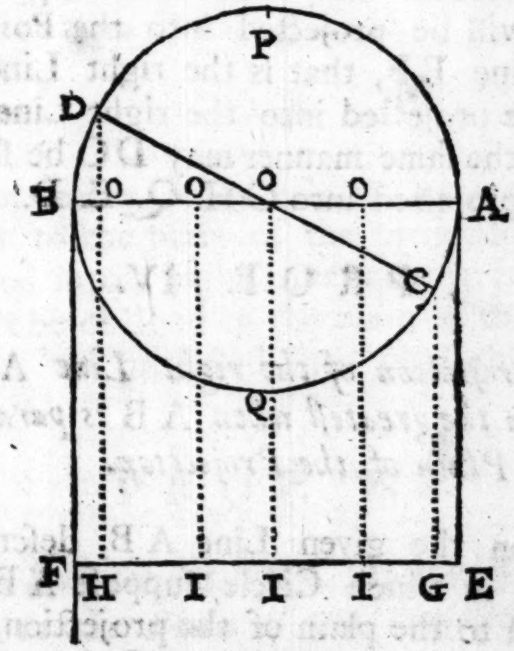
A right Line perpendicular to the Plain of the Projection, is projected into a Point, where that right Line cuts the Plain of the Projection.

This *Prop.* is evident from the firſt *Definition.*

PROP.

PROP. III.

A right Line, as A B, *not perpendicular, but either parallel or oblique to the Plain of the Projection, is projected into a right Line, as* E F, *and is always comprehended between the axtream perpendiculars* B F *and* A F.



Thro' E A, B F draw a Plain, cutting the Plain of the Projection in the right Line E F.

Since then, A E and B F are Normal (by the ſuppoſition) to the Plain of the Projection, the plain E A B F will likewiſe be Normal to it. Therefore, having taken any Number of points in the right Line A B, as O, O, O, and from theſe having drawn O I, O I, O I, at right Angles to the common Section E F, theſe will alſo (as being in the plain of E A B F) be perpendicular to the plain of the projection. Therefore all the points of the right Line A B will be projected into the Points of the Line E F, that is the right Line A B will be projected into the right Line E F. After the ſame manner may D C be ſhown to be projected into G H. Q. E. D.

PROP. IV.

The Projection of the right Line A B, *is then the greateſt when* A B *is parallel to the Plain of the Projection.*

Upon the given Line A B, deſcribe a Circle, in which Circle ſuppoſe A B firſt parallel to the plain of the projection, and then moved into the oblique Poſition D C; and draw the perpendiculars A E, B F: As likewiſe, D H and C G.

By the preceeding *Prop.* A B is projected into E F, and C D into G H. Now ſince the Angles A E F and B F E are right (by the ſuppoſition) and that A B is parallel to E F; the Angles E A B and F B A will likewiſe be right, and conſequently E A and B F will touch the Circle in B and A; therefore any other Lines drawn perpendicular to B A or E F, which is equal and parallel to B A, will fall within the perpendiculars A E and B F; and conſequently E F is greater than H G. Q. E. D.

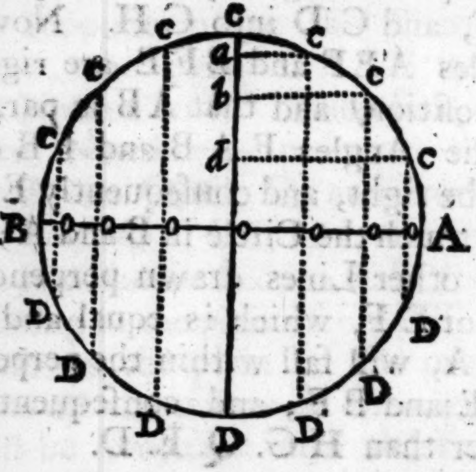
C O R.

From hence it is evident, that a Line parallel to the plain of the projection, is projected into a right Line equal to its ſelf; but if it be oblique to the plain of the Projection, 'tis projected into one which is leſs.

P R O P. V.

A plain Surface, as A B C D, *at right Angles to the Plain of the Projectiou, is projected into that right Line (as* A B*) in which it cuts the Plain of the Projection.*

In



In the given Plain, draw any Number of Lines, as C O D, C O D, *&c.* at right Angles to the common Section A B; then becauſe the given plain A B C D is perpendicular to the plain of the projection by the Hypotheſis, the right Lines C O D, C O D *&c.* perpendicular to the common Section A B, will alſo be perpendicular to the plain of the projection; therefore the right Lines C O D, C O D, *&c.* are projected into the points O, O, which are points in the common Section A B, and conſequently the whole plain A B C D is projected into the Line A B. Q. E. D.

COR. I.

Hence it is evident, That the Circle BCAD ſtanding at right Angles to the plain of the projection, which paſſes thro' its Centre, is projected into that Diameter AB, in which it cuts the plain of the projection.

COR. II.

It is likewiſe evident, That any Ark as CC is projected into OO, equal to C*a*, C*b* or C*d*, which is the right Sine of that Ark; and the complemental Ark CA is projected into OA, the verſed Sine of the ſame Ark CC.

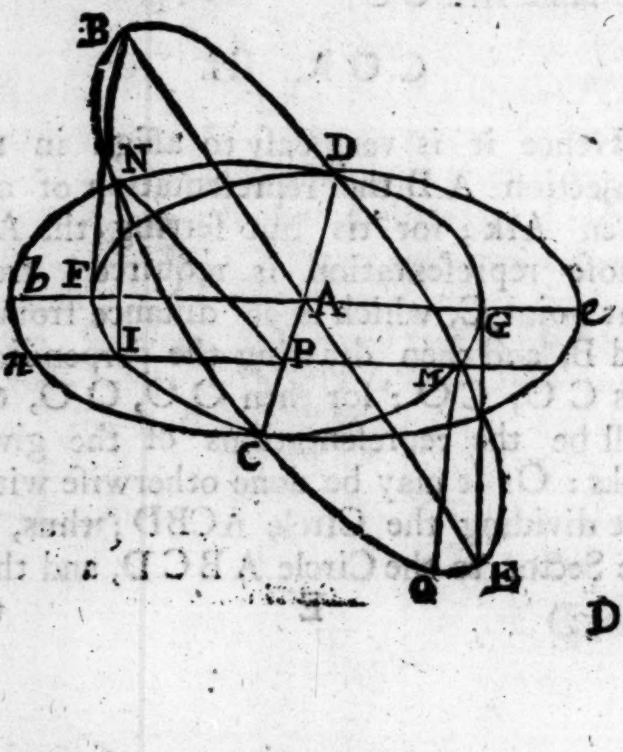
COR. III.

Hence it is very eaſy to aſſign in the projection AB the repreſentation of any given Ark; for 'tis but ſetting the Ark whoſe repreſentation is required, from that point C, which is 90° diſtance from A and B, and then drawing the perpendiculars CO, CO; for then OO, OO, *&c.* will be the repreſentations of the given Arks: Or it may be done otherwiſe without dividing the Circle ACBD; thus, fit the Sector to the Circle ABCD, and then the right Sine of any Ark ſet off from the Centre, either Way in the right Line, AB, will give the true Repreſentation of that Ark; as is evident from *Cor.* 2.

PROP. VI.

A Circle parallel to the Plain of the Projection, is projected into a Circle equal to it ſelf and a Circle oblique to the Plain of the Projection, is projected into an Ellipſis.

The firſt part of this Propoſition is evident; and that the ſecond Part is true, I thus Demonſtrate.



DEMONSTRATION.

Let the Circle ACBDE cut the plain of the projection, with its Diameter CAD, to which draw another Diameter as BAE, at right Angles. Let the projection of the Circle, made by the perpendicular Rays BF, NI, EG, OM, *&c.* be ACFDG, in which draw FAG; becauſe BF is at right Angles to the Plain of the Projection, the Plain ABF will alſo be at right Angles to it; and ſince DA is perpendicular to BA, it will likewiſe be perpendicular to FA, another Line in the ſame Plain. Moreover draw IP, parallel to FA, and NP, parallel to BA; Wherefore, ſince BA and FA are at right Angles to DAC, NP and IP will be ſo too; then becauſe BF and NI are perpendicular to the Plain of the Projection, they will alſo be perpendicular to FA and IP, Lines drawn in that Plain, and conſequently the Angles BFA and NIP are equal and Right. Laſtly, ſince NP and PI are parallel to BA and AF, the Angles BAF and NPI are alſo equal. Therefore the Triangles BFA and NIP are ſimilar, and $FA : IP :: BA : NP$ and $FA^q : IP^q :: BA^q : NP^q$: but $BA^q : NP^q ::$ Rectangle CAD : □ CPD as being equal to them (*by* 35 *e* 3); Therefore F A^q : I P^q : : Rectangle C A D : Rectangle C P D ; which being one of the properties of the Circle and Ellipſis, it is evident that the Figure A F D G C muſt be one of 'em. But it cannot be a Circle, for B E being oblique to the Plain of the Projection is projected into a Line leſs than it ſelf. (*by Cor. Prop.* 4.) but the Line F A G is the Line into which it is projected ; therefore F A G is leſs than B A E *i. e.* then C A D = B A E ; Wherefore ſince the Diameters F G and C D, of the Figure C F D G, are unequal, it cannot be a Circle, and conſequently muſt be an Ellipſis, whoſe greater Diameter is D C and its leſſer F G. Q. E. D.

COR. I.

From what has been ſaid 'tis evident, that an Ellipſis placed direct to the Eye, will be projected into an Ellipſis equal to it ſelf.

COR. II.

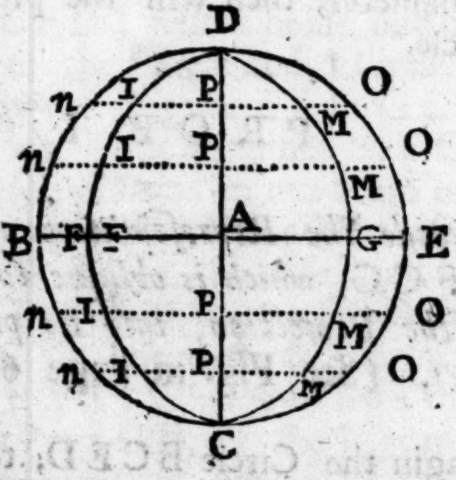
It is likewiſe evident, that an Ellipſis placed oblique to the Eye will be projected into an Ellipſis, except in one Caſe, in which it will be projected into a Circle, for if the longer Diameter be ſo much ſhortned in the Projection, that it be equal to the ſhorter Diameter, then will the projection be a Circle.

PROB. I.

To divide the Repreſentation of a Circle DFCG, *which is oblique to the Plain of the Projection, into its proper Degrees.* (See Fig. in Page *66.)*

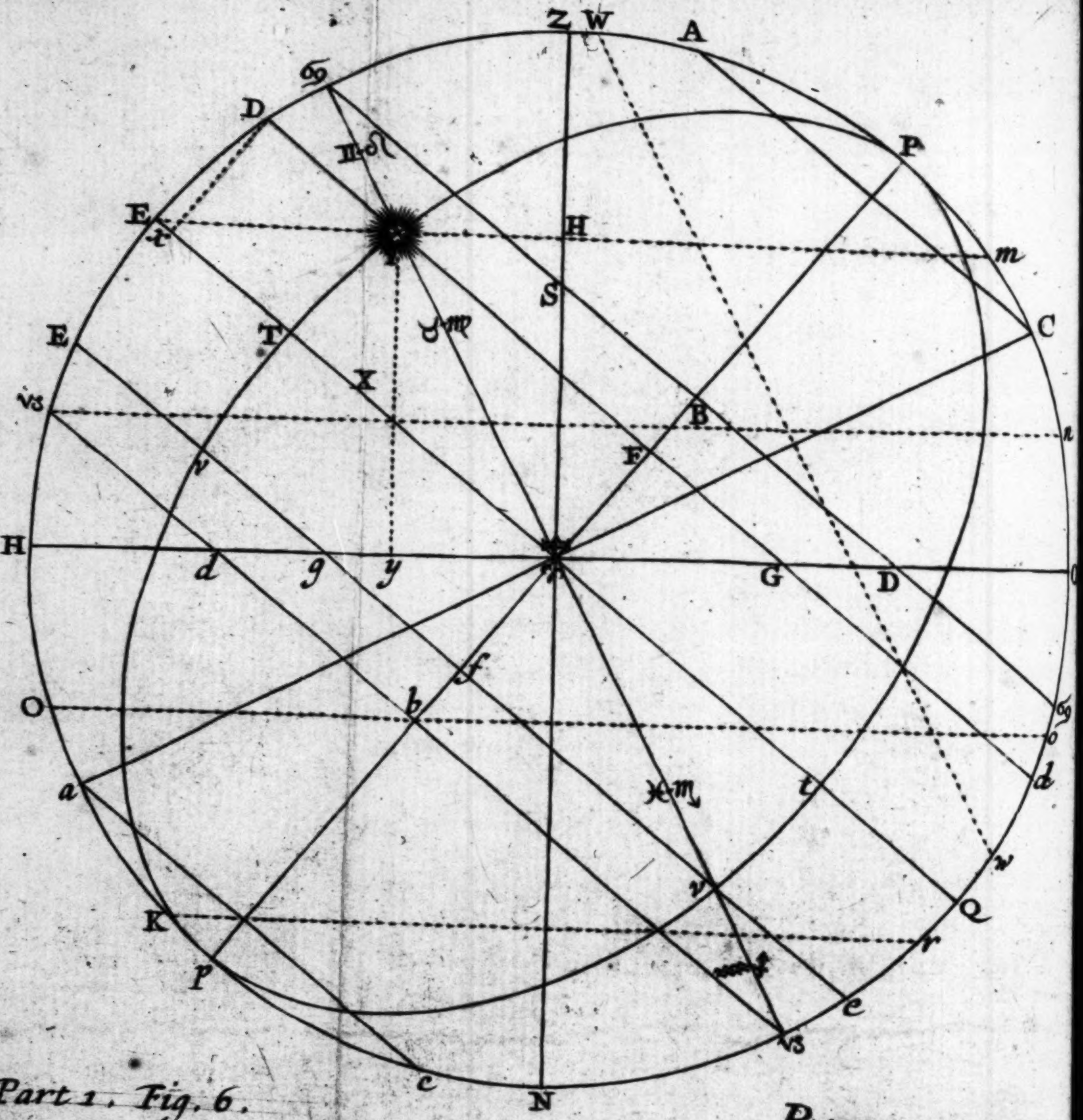
Imagin the Circle BCED, to be divided in the points B and N, and thro' thoſe points draw the Lines BA and NP perpendicular to DC, and from the ſame Lines let fall the perpendiculars BF and NI. It is plain that the Arks BN and NC are projected into the Arks FI, and IC; the ſame Thing will be evident if the Circle DBCE be turned till it fall in with the Plain of the Projection, in D*b n*C*e*, for then the right Lines BA and NP will coincide with *b*A and *n*P, and will paſs thro' the points F and I, and will remain perpendicular to DC.

If therefore a Circle be deſcribed equal to the Circle D*b*C*e*, as DBCE, and divided into Degrees in the points *n*, *n*, *O*, *O*, and from thoſe diviſions be let fall perpendiculars as *n* P, they will cut the Repreſentation D F C G, into their proper Degrees' in the Points I, I, M, M, that is I I will be the Repreſentation of as many Degrees as *n n* contains, &c. which was to be ſhown.



The Orthographick Projection of the Sphere upon the Plain of the Solſtitial Colure.

Having Demonſtrated thoſe Propoſitions which are requiſite to the Orthographick Projection of the Sphere; it will be eaſy to Project the Sphere *Orthographically* upon any great Circle. But becauſe none of the Orthographick Projections are of any great uſe, excepting that upon the *Solſtitial Colure*, I ſhall



Part 1. Fig. 6.

P. 71.

I ſhall only Project the Sphere upon this Circle for Lat. 51°. 30′. and ſhew ſome of thoſe many Uſes that may be made of this Projection, which is uſually called the *Anatemma.*

PROB. II.

To Project the Equinoctial Colure, the Horizon for any Latitude, the Equator, Tropicks, Polar Circles, Ecliptick, Parallels of Latitude, Parallels of Declination, and Almacanters or Parallels of Altitude.

Since all theſe Circles are at Right Angles to the Solſtitial Colure which is the Plain of the Projection, *by Prop.* 5, they will all be Projected into right Lines, or into their own Diameters.

Draw the Circles H N O Z repreſenting the Solſtitial Colure, and H O repreſenting the Horizon of *London*: To which at right Angles draw Z N which will repreſent the Azimuth of Eaſt and Weſt.

Set 38°. 30′ (taken from the Chords) from H to E, and draw the Diameter E Q: Which is the Repreſentation of the *Æquator* for our Lat. 51°. 30′; and which Croſs at right Angles with the Diameter P *p*, which will be the Axis of the World.

The Chord of 23°. 30′ (*the Sector being fitted to the Radius of that Limb, which I all along ſuppoſe*) Set both Ways, from E and Q, the two extream Points of the Equator, will give the Points ♋ ♑ and ♋ ♑; thro' which Points draw the right Lines ♋ ♋ and ♑ ♑, and they will be the two *Tropicks*; the firſt the *Tropick* of *Cancer*, and the latter that of *Capricorn*.

From P ſet the Chord of 23°. 30′ to A and C; as likewiſe from *p* to *a* and *c*; and draw the two Lines A C and *a c*, which will repreſent the Polar Circles, the former the *Arctick*, and the latter the *Antarctick* Circle.

Through ♋ ♑ draw the Diameter ♋ ♈ ♑, which will be the Projection of the Ecliptick, and C *a* at right Angles to it, which will be the Axis of the Ecliptick.

Parallel to the Ecliptick and at right Angles to its Axis, draw any Number of right Lines, and thoſe Lines will be parallels of Latitude of the Stars: And they may be drawn at 10°. 20°. 30°, &c, Diſtance from the Ecliptick, if the Chords of 10°. 20°. 30°, &c. be ſet from ♋ and ♑ either Way, towards P or *p* in the Limb, for they will determine the points, through which if right

Lines be drawn, they will repreſent the Circles required, or the parallels of Latitude 10°. 20°. 30°, *&c.* Diſtant from the Ecliptick ; of theſe there is but only one drawn, *viz.* W *w*, becauſe the Diagram ſhould not be crouded.

After the ſame Manner may any Number of parallels of Declination, and Almacanters be drawn, the former parallel to the Equator E Q, the latter to the Horizon H O.

PROB. III.

To divide the Repreſentation of the Ecliptick, or any other great Circle, which is at right Angles to the Plain of the Projection, into its proper Degrees.

This may be done by a Line of *Sines*, beginning from the Centre ; as is evident from *Cor.* 3. *Prop.* 5. So that the Sine of 30°. ſet from the Centre, in the Line ♋ ♑, will determine the Repreſentation of ſo many Degrees there : Likewiſe, the Sine of *60*°. ſet both Ways from the Centre, will determine the Repreſentation of *60*°. and if at the Centre be put ♈ and ♎ ; at the end of 30°. on each ſide the Centre be put ♉ ♍ and ♏ ♓ : at the End of *60*°. ♊ ♌, and ♐ ♒ : and at the End of the Sine of 90°. or Radius ♋ on the North, and ♑ on the South; then will the Repreſentation of the Ecliptick be divided into its proper Signs. And after the ſame Manner may the Repreſentation of any other right Circle be divided into its proper Degrees, from a Line of Sines.

Any leſſer Circle, which is at right Angles to the Plain of the Projection, as ♋ ♋; may be divided after the ſame Manner, by fitting the Sector to the Radius ♋ B, and then taking any Portion of it, as B S, beginning from B to its Centre, and applying it parallelwiſe to the Lines of Sines on the Sector; for you then will have the Number of Degrees which the Portion B S repreſents.

PROB. IV.

To Project the Hour-Circles, Azimuths, and Circles of Longitude.

Divide the Equator by the foregoing *Prob.* and then through every Degree or every 10 Degrees, as you pleaſe, you may draw the the Hour-Circles, (which will be Ellipſes in this Projection, by *Prop. 6.)* after this Manner. Take two Points, in the Equator as T and *t*, at equal Diſtances each Way from the Centre ; Suppoſe 42°, then draw a ſufficient Number of Lines parallel to the Equator ; as D *d*, E *e*, &c. and then ſay, as E ♈ : T ♈ : : D F : F Y : : *f* E : *f v*, &c. which will find Y and *v* points thro' which the Ellipſis muſt paſs ; and after the ſame Mnner an Infinite Number of points may be found, thro' which the Ellipſis may be drawn.

N. B. Azimuths and Circles of Longitude may be deſcribed after the ſame Way.

Wherefore the Diagram need not be crouded with them, which would too much confound the Eye.

PROB. V.

To divide the Hour-Circles, Azimuths, and Circles of Longitude, into their proper Degrees.

This may be done by *Prob.* 1. for inſtance, the Hour-Circle P T *p* may be divided into its proper Degrees, by drawing any Number of Parallels to E Q ; thus the Number of Degrees, which T ☉ repreſents, may eaſily be known by drawing D *d* parallel to E Q through the point ☉, and then meaſuring E D on the Line of Chords (the Sector being firſt ſet to the Radius of the Limb or primitive Circle) which will give the Number of Degrees which T ☉ repreſents; and in like Manner may any other Portion of an oblique Circle be meaſured.

PROB. VI.

To Project the Stars Orthographically.

Before this *Problem* can be ſolved, you muſt know the Diſtance of the Star you are to project from the Equinoctial, and from the Ecliptick, *(i. e.)* its Declination and Latitude; ſuppoſe the Declination of a Star be 17°. N. and its Lat. 30°. N. and it were required to project this Star Orthographically: Firſt ſet the Chord of 17°. from E to D, and draw the Line D F *d*, and then ſet the Chord of 30°. from ♋ to W, and draw the prickt Line W *w*, parallel to the Ecliptick, which repreſents the Stars parallel of Latitude, and it will cut its parallel of Declination D *d* in the point required, for the Star will be projected there, where theſe two Lines interſect one another.

PROP. VII.

Some of the Uſes that may be made of this Projection.

I. By this Projection, may be gained the Suns preſent Declination at any Time. Suppoſe the Suns Declination were required for the 28th of *July*, firſt find the Sign and Degree of the Ecliptick he is then in, which is the 15°. of *Leo* ; then conſider how far the 15° of *Leo* is diſtant from the next Equinoctial Point, which is *Libra*, and you'll find it to be 45°. which taken from the Sines, and ſet from the Centre in the Ecliptick, will give the point ☉, *viz.* the Suns Place in the Ecliptick at that Time ; then through ☉, parallel to the Equinoctial E Q, draw the Line D F *d*, which will be the parallel that the Sun moves in that Day ; and E D applied to the Chords, or D x = ♈ F applied to the Sines, will give the Suns Declination at that Time, *viz.* 16°. 30′.

II. The Suns Amplitude, and conſequently the length of the Day may be gained by applying ♈ G to the Line of Sines, and then converting thoſe Degrees into Time ; thus, ♈ G applied to the Sines will give 26°. the Suns Amplitude, which converted into Time, will give 1^{h}. + ¾ of an Hour nearly ; So that the Sun riſes that Day near ¾ of an Hour before 5 a-Clock, and ſets near ¾ after 7. Wherefore the length of the Day at that Time will be near 15 Hours and a half, and conſequently the length of the Night will be near 8 Hours and a half.

III. The Suns right Aſcenſion for that Day may be had thus, apply the Ark ♈ T of the Equator to the Line of Sines, which you will find 42°. then ſubtract 42°. from 90°. and there will remain 48°. which added to 90°. becauſe the Sun is paſt one Quadrant of the Ecliptick, gives 138°, the Suns right Aſcenſion at that Time.

IV. That point in the Parallel ~~in~~ which the Sun is at any aſſigned time of the Day, may be had thus : Suppoſe the Time aſſigned be 8 a-Clock in the Morning, conſider the Diſtance of 8 a-Clock from 6, which is 2 Hours, which converted into Degrees gives 30°. the Sine whereof ſet (by the Sector) from ♈ to X, and then ſay, as E ♈ is to ♈ X, ſo is D F to a fourth, which ſet from F, will give the Point in which the Sun is at that Time, *viz.* 8 a-Clock in the Morning.

And on the contrary, having given the point in the Parallel in which the Sun is, the Time of the Day may be eaſily found, by working backwards, and ſaying, as D F is the Diſtance from F to the point given, ſo is E ♈ to ♈ X: Then applying ♈ X to the Sines, and converting the Degrees into Time, you will have the Time of the Day.

V. Suppoſe *July* the 28*th*, at 9 a-Clock in the Morning you would know how high the Sun is, or its Altitude above the Horizon: From the Hour given find the point in the parallel for that Day by the laſt, which let be ☉, and through that point draw E H *m* parallel to the Horizon, cutting the Limb in E and *m*, then apply E H to the Chords, and it will give you his Altitude at that Time.

Or ☉ *y* or H ♈ which is equal to it, being applied to the Sines, will do the ſame thing. Or on the contrary, you may find the Hour of the Day by having given his Altitude.

There are a great many more Uſes belonging to this Projection beſides thoſe which I have here mentioned, but theſe, together with their Reaſons (which for Brevities ſake I have here ommitted, becauſe they are evident from the foregoing Projection) being thoroughly underſtood, the Reader will eaſily make out the reſt him ſelf.

THE

THE

Stereographick Projection

OF THE

SPHERE.

PART II.

DEFINITIONS.

1. THE *Pole* of any Circle, is a point upon the Surface of the Sphere, from whence all right Lines that can be drawn to the circumference of that Circle are equal.

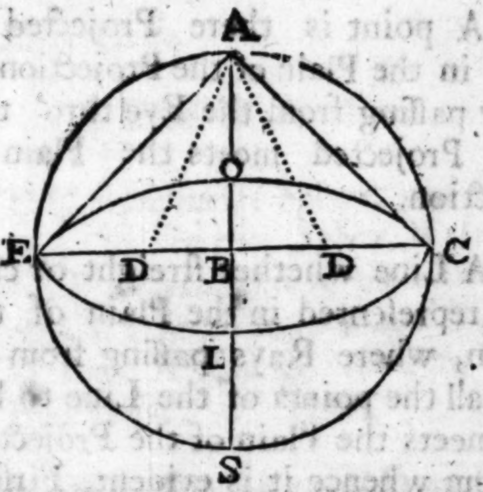
N. B There is another Definition of the Pole of a Circle, and that a very true one, *viz. That 'tis a point in the Axis* 90°. *above the plain of the Circle*; but this is better for our preſent Purpoſe.

2. The *Plain of the Projection*, is the plain of a great Circle infinitely produced every Way, and perpendicularly oppoſed to the Eye placed in the Surface of the Sphere.

3. That great Circle (whoſe plain being produced every way, makes the plain of the Projection) is called the *Primitive Circle.*

Let the Eye be placed in any point in the Surface of the Sphere, as at A from which point through the Centre of the Sphere, draw the Diameter ABS, to which ſuppoſe an infinite Plain to be drawn at right Angles, cutting the Sphere in the Circle C L E O, an infinite Plain ſo drawn, is called the Plain of the Projection; and the Circle C L E O, is called the Primitive Circle.

Hence



Hence it is evident; Firſt, that the point A, where the Eye is placed, is the Pole of the primitive Circle C L E O, (by *Lem.* 1. *Theod. Spher.*) which is apparent alſo from its being 90°. diſtant from the plain; and ſo poſited that Lines drawn from it to the Circular Plain of the Projection are equal.

And Secondly, that all great Circles which paſs thro' the Eye at A as A C S E are at right Angles to the Primitive ; for all of them paſs through the Diameter A B S, which is at right Angles with the ſaid Circle, and conſequently will be perpendicular to the Primitive. (by 18. *e.*11.)

4. A point is there Projected into a Point in the Plain of the Projection, where a Ray paſſing from the Eye thro' the point to be Projected meets the Plain of the Projection.

5. A Line whether ſtreight or crooked is there repreſented in the Plain of the Projection, where Rays paſſing from the Eye thro' all the points of the Line to be projected meets the Plain of the Projection.

From whence it is evident, Firſt, that if a ſtreight Line paſs thro' the Eye, its Repreſentation will be but a point in the Plain of the Projection; for one and the ſame Ray will paſs thro' all the points of that Line, (as in *Fig. preced.*) C L E O being the primitive Circle, the Repreſentation of the Line A S paſſing thro' the Eye at A will be B, *viz.* that point where it cuts the primitive Circle, or in caſe it fall without the Primitive, where it cuts the Plain of the Projection.

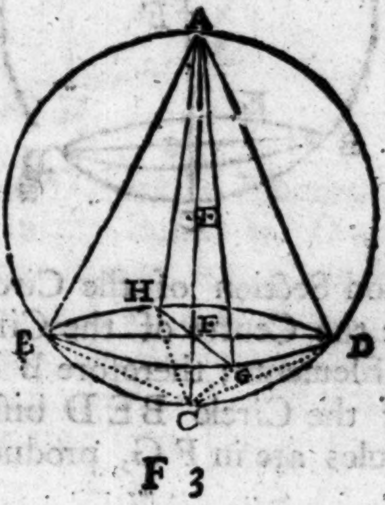
And Secondly, that the Projection of a Circle paſſing thro' the Eye will be an infinite right Line, for in *Fig.* 1. the Projection of the Semi-circle E S C oppoſite to the Eye, will be E C and the projection of that Semi-circle in which the Eye is placed, will fall in the Line E C produced infinitely both Ways from E and C.

VI. A Circle paſſing thro' the Eye is called a *Right Circle*.

LEMMA I.

Let B A D C *repreſent the Sphere, and let* B G D H *be a Circle in that Sphere, to the Centre* E *draw* E F *perpendicular to the Circle* B G D H, *and let it be produced; and I ſay, it will fall upon* A *and* C, *the Poles of the Circle* B H D G.

Draw the two Diameters B D and G H, and the Lines A B, A D, A H, A G; now ſince the Lines F B, F D, F G, F H are all equal to one another, being Radii of the ſame Circle; and the Angles, A F B, A F D, A F G, A F H being Right, their Subten-

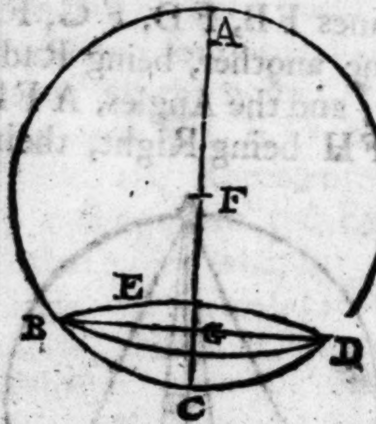


ſes A B, A D, A G, A H will be equal alſo Therefore A is the Pole of the Circle B G D H by *Defi.* 1, and in like Manner may C be ſhewn to be the other Pole. Q. E. D.

LEMMA II.

If A B C D *a great Circle of the Sphere, cut any other Circle, as* B E D, *at right Angles, it biſſects that Circle, and paſſes thro' its Poles.*

From F the Centre of the Sphere, or the great Circle A B C D, draw F G at right Angles to the Circle B E D, meeting B D

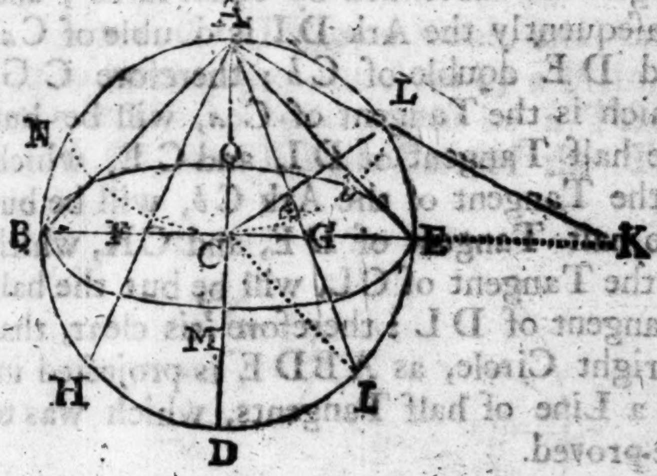


the common Section of the Circles, in G, G will be the Centre of the Circle B E D from the Elements; therefore B D is a Diameter of the Circle B E D biſſecting it; and its Poles are in F G, produced by the firſt *Lemma* and conſequently in the Circle ABCD. Q. E. D.

PROP. I.

A right Circle is projected into a Line of half Tangents.

By *Defini.* 5, with its *Cor.* The Semicircle BDE will be projected into BE; divide the Semi-circle BDE into a Number of parts; ſuppoſe 4, in the Points H, D and I, and then from the Eye at A to theſe Diviſions, draw the Lines A H, A D and A I, which Lines will cut the Repreſentation B E in the points F, C and G; Let one of the Lines as A D paſs thro' the Centre C, and con-



ſequently be at right Angles to B E; with the Radius A C and from the Centre A deſcribe the Circle L C N, B E will touch the Circle at the point C, and C G will be the Tangent of the Ark C *a* and C E the Tangent of the Ark C *b*; but C *a* is equal to half the Ark D I, and the Ark C *b* is equal to half the Ark D E; for the Angle D A I is equal to half D C I (from the Elements) but in equal Circles equal Angles are ſubtended by equal Arks; and conſequently the Ark D I is double of C *a*, and D E double of C *b*; therefore C G, which is the Tangent of C *a*, will be but the half Tangent of D I, and C E, which is the Tangent of the Ark C *b*, will be but the half Tangent of D E, and C K, which is the Tangent of C L, will be but the half Tangent of D L; therefore 'tis clear, that a right Circle, as A B D E is projected into a Line of half Tangents, which was to be proved.

7. A Circle not right to the Eye, is there repreſented in the Plain of the Projection, where Rays paſſing from the Eye thro' all the points of the Circle, meet the Plain of the Projection; or where a Cone of Rays, having its Vertex in the Eye, and paſſing through all the points of the Circle to

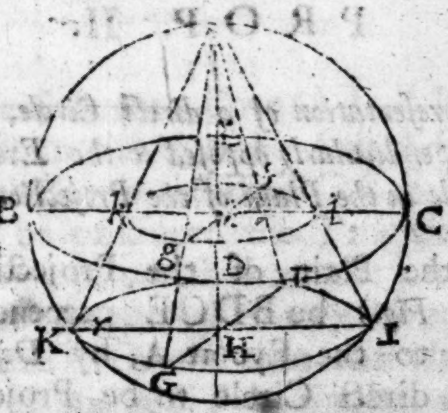
be projected, meets the Plain of the Projection.

8. A Circle perpendicularly oppoſed to the Eye is called a *Direct Circle.*

LEMMA III.

If a Cone be cut by a Plain parallel to the Baſe, the Section will be a Circle.

Let A K G I F be a Cone, and imagin it to be cut thro' the Axis by a Plain, and the Section produced will be a Triangle as A K I, and K I the Baſe of the Triangle will be a Diameter of the Baſe of the Cone K G I F; draw *k i* parallel to K I, and upon *ki* imagin a plain erected perpendicular cutting the Cone in *k g i f*; I ſay, the



the Section $kgif$ will be a Circle for ki being parallel to KI, the Triangles Ahk and AHK will be ſimilar, conſequently $Ah : hk :: AH : HK$; then imagin another Triangular Section as AGF, in which GF and gf will be parallel, and conſequently the Triangles AHG and Ahg will be ſimilar, as likewiſe AHF and Ahf; wherefore $Ah : hg :: AH : HG$, and $Ah : hf :: AH : HF$; conſequently *ex æquo* $KH : HG :: gh : hk$, and $hg : hf :: HG : HF$. KH, HG and HF are all equal, being Radii of the Baſe of the Cone; therefore kh, hg and hf are equal alſo, and conſequently the Section $kgif$ is a Circle. Q.E.D.

PROP. II.

The Repreſentation of a direct Circle, or a Circle perpendicularly oppoſed to the Eye, will be a Circle in the Plain of the Projection.

Let the Plain of the Projection (ſee preced. *Fig.*) be BDCE perpendicularly oppoſed to the Eye at A, by *Def.* 2. and let the direct Circle to be Projected be KGIF, perpendicularly likewiſe oppoſed to the Eye at A; Wherefore the two Plains BDCE and KGIF will be parallel to one another, and conſequently the Cone of

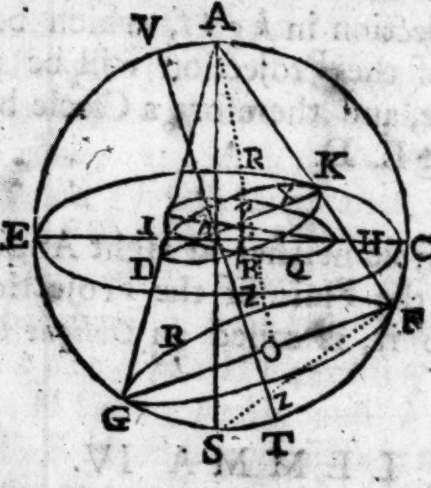
Rays A K G I F will be cut by the Plain of the Projection in *k g i f,* which being in the Plain of the Projection will be parallel to the Baſe, and therefore a Circle by *Lemma 3.* Q. E. D.

9. A Circle neither at right Angles, nor parallel to the Plain of the Projection, but enclined to it, is called an *Oblique Circle.*

LEMMA IV.

The Section of a ſcalenous Cone, cut ſub-contrarily to its Baſe, will be a Circle.

Let the ſcalenous Cone A G Z F R be cut by a Plain through the Axis A O, and at right Angles to the Baſe G Z F R, and there will be produced the Triangular Section A G F; in which draw the Line I H, ſo that the Angle A I H may be equal to the Angle A F G, and then will the Triangle A I H and A G F be ſimilar, and are ſaid to be ſub-contrarily poſited; upon the Line I H, at right Angles to the Triangle A G F, imagin a plain to be erected, cutting the Cone in I B H; I ſay, the Section



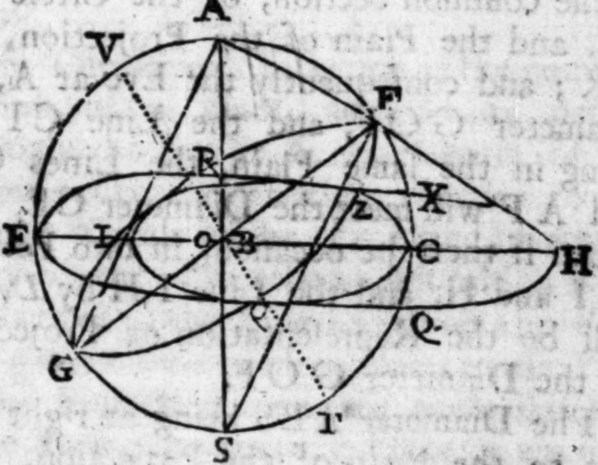
on I B H will be a Circle take P a Point any where in I B H, and draw B P perpendicular to I H; through P draw D K parallel to G F, and then if the Cone be cut through D K by a plain perpendicular to A G F, the Section D B K will be a Circle by *Lemma* 1. And B P being at right Angles to D K, as well as I H, ſince both D K and I H are Lines in the Triangle A G F, D P × P K will be equal to B P *q*; but the Triangles I P D and K P H are ſimilar, for the Angles at P are equal, and A I H is equal to A K D being ſub-contrarily poſited, and conſequently their Complements D I P and P K H are equal alſo; therefore D P : P I : : P H : P K, and D P × P K = P I × P H. Wherefore P I × P H = P*p q*,

= P *p q*, and conſequently B is at the Circumference of a Circle, of which Circle I H is Diameter, wherefore I B H is a Circle. Q. E. D.

PROP. III.

The Repreſentation of a Circle placed oblique to the Eye, will be a Circle in the Plain of the Projection.

Take any Circle inclined to the Plain of the Projection, whether it be a greater Circle or a leſſer, it matters not, for the ſame



Demonſtration will ſerve for all : (ſee preced. *Fig.*) and let the Cone of Rays, under which the Eye ſees this Circle be AFZGA, cutting the Plain of the Projection CZER in HQIX. By *Def.* 7. HQIX will be the Repreſentation of the inclined or oblique Circle FRGZ.

Let the Sphere be cut by a Plain, paſſing through the Eye at A, the Centre B, and at right Angles to the Plain of the Oblique Circle FRGZ; it will paſs thro' its Centre O (by *Lem.* 2.) and likewiſe its Poles T and V, and conſequently FG will be a Diameter, and ACSE a great Circle of the Sphere; in which Circle, draw the Lines ABS, FS, AF, AG and AO, which AO will be the Axis of the Cone AFZGA, and the Line CHIE is the common Section, of the Circle ACSE, and the Plain of the Projection CZER; and conſequently the Eye at A, the Diameter GOF, and the Line CIHE being in the ſame Plain, the Lines GA and AF will meet the Diameter CE, produced if their be occaſion, in two Points; as I and H, and the Line IH by *Def.* 5. will be the Repreſentation or Projection of the Diameter GOF.

The Diameter ABS being at right Angles to the Plain of the Projection, will be at right Angles to HI a Line in that Plain. Therefore ABH being a right Angle, it will be equal to AFS, and the

Triangle at A being common, the Angle A H B will be equal to ASF, and conſequently the Triangle A H B is ſimilar to the Triangle A F S; but the Angle A S F is equal to the Angle A G F (from the Elements) therefore the Angle A H B is equal to the Angle A G F, and the Angle at A being common to the two Triangles I A H and G A F, theſe two Triangles will likewiſe be ſimilar, but poſited ſub-contrarily; therefore the Section H Q I, made by the Plain of the Projection C Z E R, paſſing thro' I A, will be a Circle by *Lemma* 2. Q. E. D.

DEFINITION.

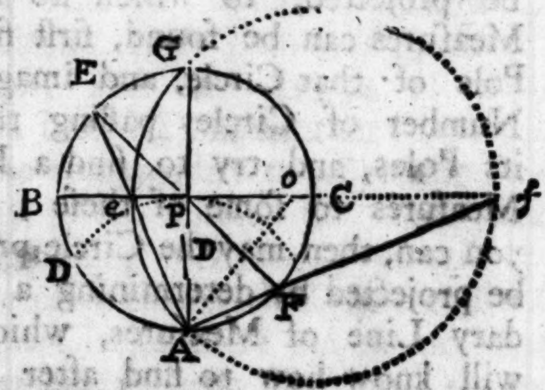
10. The Line C E, which is the common Section of a Plain, as A S C E, paſſing through the Eye at A, the Centre of the Primitive B, and at right Angles to the oblique Circle to be Projected, with the Plain of the Projection; or which is the ſame Thing, the Line in which the Plain of a great Circle perpendicular to the Plain of the Projection, and the oblique Circle to be Projected, cuts the Plain of the Projection; I ſay, ſuch a Line is called a Line of *Meaſure.*

N. B. In the Stereographick Projection of the Sphere, 'tis of abſolute Neceſſity that the *Line* of *Meaſures* be determin'd, before you can Project any oblique Circle; for 'tis in the Line of Meaſures, that the Centre of every oblique Circle is found, not in one and the ſame Line of Meaſures, but in a Line of Meaſures proper to that oblique Circle you have a Mind to project; ſo that 'twill be very Neceſſary to have a thorow Knowledge of the Sphere, before you meddle with Sperick Projection: That you know all its Circles, their Diſtances from one another, and the Angles of thoſe which interſect one another; that you may be able readily to aſſign the Pole of any given Circle, *&c.* and theſe Things being underſtood, you will eaſily perceive that with Reſpect to ſome oblique Circles there can be no Line of Meaſeres properly ſpeaking, *i.e.* there cannot be found a Line which ſhall agree to the *Def.* 10. becauſe no great Circle can be drawn at right Angles both to the oblique Circle, and the Plain of the Projection: As in the Projection of the Hour Circles upon the Plain of the Horizon for any aſſigned Latitude leſs than 90°, 'tis certain, no great Circles will be perpendicular both to the Plain of the Projection, and any of the Hour Circles, excepting that of 6; ſo that in the Projection of all the reſt, we muſt have Recourſe to another Artifice, and determine another Line in which all their Centers ſhall be found, and which Line I ſhall call a *ſecondary Line of Meaſures*, and the Way of finding ſuch a Line I ſhall ſhew by and by. But in caſe any oblique Circle be propoſed to be projected, to which no Line of Meaſures can be found, firſt find the Pole of that Circle, and imagin any Number of Circles paſſing through its Poles, and try to find a Line of Meaſures to ſome of theſe; and if you can, then may the Circle propoſed be projected by determining a ſecundary Line of Meaſures, which you will know how to find after I have premiſed theſe four Propoſitions, by the help of which it will be eaſy, by Scale and Compaſs, to project the Sphere upon the Plain of any great Circle.

PROP. IV.

If a great Circle be to be projected upon the Plain of another great Circle, its Centre ſhall lie in the Line of Meaſures, diſtant from the Centre of the Primitive by the Tangent of its Elevation above the Plain of the Primitive.

Draw the Circle A B G C, which imagin to repreſent the Primitive, in whoſe under Pole let the Eye be placed ; and let A *e* G be the oblique Circle to be projected, elevated above the Primitive any number of Degrees : Imagin a Plain paſſing thro' the Eye perpendicular to the primitive and oblique Circle, then there will be produced a Section, which may be repreſented by A B G C, cutting the Plain of the Projection



ction in B C, and the oblique Circle A *e* G in E F; the Place of the Eye in this Section is at A. Draw the Lines A E, and AF producing them 'till they meet B C produced in *e* and *f*; *e f* will be the Repreſentation of the Diameter E F; and conſequently being biſſected in *o*, the Circle A *e* G *f* may be deſcribed, which is the Repreſentation of the propoſed oblique Circle, the Eye being placed in the under Pole of the Primitive A B G C. Now I am to prove, that *o* the Centre of the projected oblique Circle, is diſtant from P the Centre of the Primitive, by the Tangent of its Elevation above the Primitive; *i. e.* that *o* P is equal to the Tangent of the Ark B E.

From the Centre A, with the Radius A P, deſcribe the Ark D P F; the Angle E A G is equal to one half of E P G: The Angle E A G is likewiſe equal to A *f* P, and A *f* P is equal to $\frac{1}{2}$ P *o* A; therefore the Angles P *o* A and E P G are equal; and conſequently, their Complements E P B and P A O will be equal alſo. But in equal Circles equal Angles are ſubtended by equal Arks; therefore the Arks B E and P D are equal, but P O is the Tangent of the Ark P D, and therefore equal to the Tangent of the Ark B E, which is the Elevation of the oblique Circle above the Plain of the Projection. Q. E. D.

But here it may be objected, That I uſe the Circle A *e* G both for the oblique Circle to be projected, and for its Repreſentation in the Plain of the Projection; which I own; nor can it be otherwiſe, for a Circle elevated above the Primitive cannot be drawn otherwiſe than as it is repreſented in the Plain of the Projection.

COR.

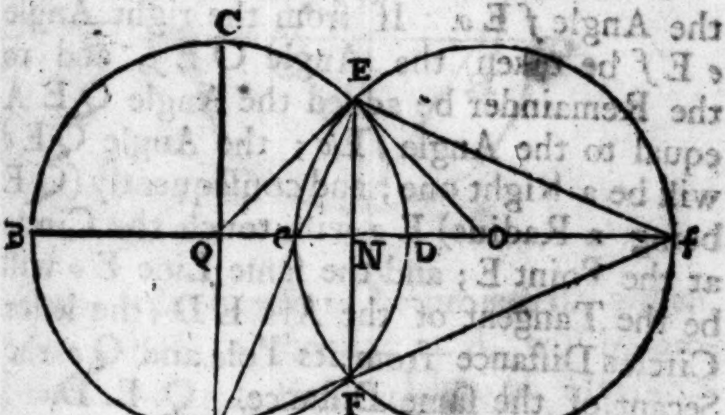
Hence it is evident, *e o* and A *o* being equal, that the Secant of the Elevation of the oblique Circle above the Primitive being ſet from *e*, the projected Extremity of the Diameter E F in the Line of Meaſures will find O the Centre of the oblique Circle; or, on the contrary, having found O the Centre of the oblique Circle, by ſetting the Tangent of its Elevation above the Primitive from P to O, then the Secant of the ſame Elevation ſet from O, will find *e* a point thro' which it muſt paſs.

PROP.

PROP. V.

If a leſſer Circle whoſe Poles lie in the Plain of the Projection were to be projected ; the Centre of its Repreſentation ſhall be in the Line of Meaſures, diſtant from the Centre of the Primitive by the Secant of that leſſer Circle's diſtance from its Pole ; and its Semidiameter or Radius ſhall be equal to the Tangent of that Diſtance.

Suppoſe BD the Line of Meaſures, and EF a Diameter of the leſſer Circle to be projected, both determined by the Section of a Plain, as above directed ; the Poles of the leſſer Circle E F will be B and D (*by Lemma* 2d) :



Draw

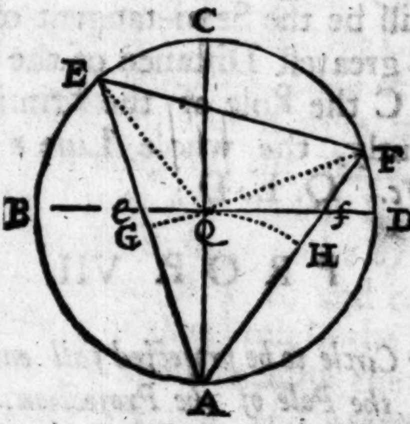
Draw the Lines A *e* E and A F *f*; the Diameter E F, with reſpect to the Eye at A, will be projected into *e f*, which, being biſſected, will give *o* the Centre of the Projection of the leſſer Circle E F. Now I am to prove, That *o* the Centre of the projected leſſer Circle, is diſtant from Q the Centre of the Primitive by the Secant of the Ark E D, the Diſtance of the ſaid leſſer Circle from its Pole D. Draw the Lines Q E, E *o* and E *f*; the Triangle *e* E *f* is Right-angled at E, and is divided into two ſimilar Triangles by the Diameter E F; therefore the Angle *e* E N is equal to the Angle E *f* N, which is equal to *f* E *o*: But the Angle *e* E N is equal to the Angle E A Q, which is equal to Q E A.

Therefore the Angle Q E A is equal to the Angle *f* E *o*. If from the right Angle *e* E *f* be taken the Angle O E *f*, and to the Remainder be added the Angle Q E A equal to the Angle *f* E *o*; the Angle Q E *o* will be a Right one; and conſequently (Q E being a Radius) E *o* will touch the Circle at the Point E; and the ſame Line E *o* will be the Tangent of the Ark E D (the leſſer Circles Diſtance from its Pole and Q *o* the Secant of the ſame Diſtance. Q. E. D.

PROP. VI.

If a leſſer Circle were to be projected, whoſe Poles lie not in the Plain of the Projection; its Diameter in the Projection, if it fall on each ſide the Pole of the Primitive, will be equal to the Sum of the half Tangents of its greateſt and neareſt Diſtance from the Pole of the Primitive, ſet each Way from the Centre of the Primitive in the Line of Meaſures.

Things being ſuppoſed as above, let B D be the Line of Meaſures; E F a Diameter

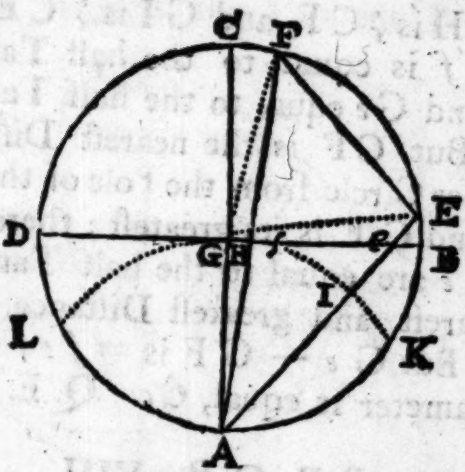


of the leſſer Circle to be projected; which, with reſpect to the Eye at A, will be projected into *e f* in the Line of Meaſures. Upon the Centre A, and with the Radius A Q deſcribe the Ark G Q H, which Ark the line B D will touch at the point Q, and *e* Q will be the Tangent of the Ark GQ, and Q *f* the Tangent of the Ark QH; but G Q is equal to $\frac{1}{2}$ E C, and Q H equal to $\frac{1}{2}$ C F, becauſe the Angle E A C is equal to $\frac{1}{2}$ E Q C, and F A C equal to $\frac{1}{2}$ F Q C; but in equal Circles equal Angles are ſubtended by equal Arks; and conſequently E C is double of G Q, and C F double of Q H; therefore *e* Q, which is the Tangent of the Ark G Q, will be the Semi-tangent of the Ark E C, the neareſt Diſtance of the Circle E F from C the Pole of the Primitive; and Q *f* the Tangent of the Ark Q H, will be the Semi-tangent of the Ark C F, the greateſt Diſtance of the leſſer Circle from C the Pole of the Primitive; and conſequently, the whole Line *e f* will be equal, *&c.* Q. E. D.

PROP. VII.

If a leſſer Circle to be projected fall entirely on one ſide of the Pole of the Projection, and do not encompaſs it as in the laſt Prop. then will its Diameter be equal to the Difference of the half Tangents of its greateſt and neareſt Diſtance from the Pole of the Primitive, ſet off from the Centre of the Primitive one and the ſame way in the Line of Meaſures.

Suppoſe

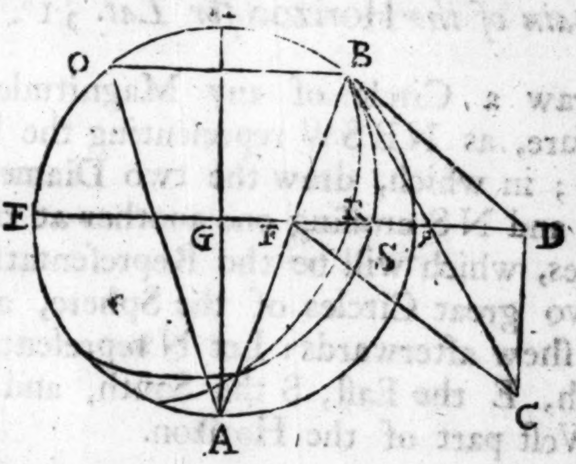


Suppoſe A D C B a great Circle of the Sphere, cutting the primitive Circle in DB (which is therefore the Line of Meaſures) and the leſſer oblique Circle in E F: Let E F fall on one and the ſame ſide of C: Draw the Lines A F and A E cutting the Line of Meaſures in *f* and *e*; *fe* therefore is the Projection of F E, and conſequently the Diameter of the Circle in the Projection: But *fe* is the Difference of the half Tangents of its greateſt and neareſt Diſtance from C the Pole of the Primitive; which I thus prove: upon the Centre A and with the Radius A G deſcribe the Ark L G K; which Ark, the Line BD, (being at right Angles to A G) will touch at the point G; and G *f* will be the Tangent of the Ark G H, G H, and G *e* the Tangent of the Ark GI: But G H is $\frac{1}{2}$ C F, and G I is $\frac{1}{2}$ C E; therefore G *f* is equal to the half Tangent of C F, and G*e* equal to the half Tangent of C E: But C F is the neareſt Diſtance of the leſſer Circle from the Pole of the Primitive, and C E is its greateſt; therefore G *f* and G *e* are equal to the half Tangents of its neareſt and greateſt Diſtance from its Pole: But G *e* — G F is = *f e*; therefore its Diameter is equal, &c. Q. E. D.

PROP. VIII.

In the Stereographick Projection, the Angles made by the Circles on the Surface of the Sphere, are equal to the Angles made by their Repreſentatives in the Plain of the Projection.

Suppoſe the Eye at A projecting the Angle R B S upon the Primitive E F S; draw B D a Tangent to the Circle B S, and B C a Tangent to the Circle B R, in B the point of their Interſection, which Tangent will make the ſame Angles that the Planes of the Circles do; the Plain E F S; and likewiſe the Plain made by the Tangents B D and B C, are both perpendicular to the Plain of the Circle B Q A, and their common Section D C will therefore be perpendicular to the Line E F produced. The Eye at A projects the Tangent B D into



FD, and the Tangent BC into FC. Now I am to prove that the Angle C B D made by the Tangents to theſe Circles on the Sphere is equal to the Angle C F D made by the projected Tangents on the Plain of the Projection, Draw B Q parallel to E F and joyn A Q ; the Angle D B A is equal to the Angle A Q B, equal to A B Q, equal to B F D, equal to D B F ; therefore B D is equal to D F : Then in the Triangles C D F, C D B, C D, D F are equal to C D, D B, and the Angles C D F and C D B both right, therefore C F D is equal to C B D. Q. E. D.

The Projection of the Sphere, upon the Plain of the Horizon *for Lat.* 51°. 30′.

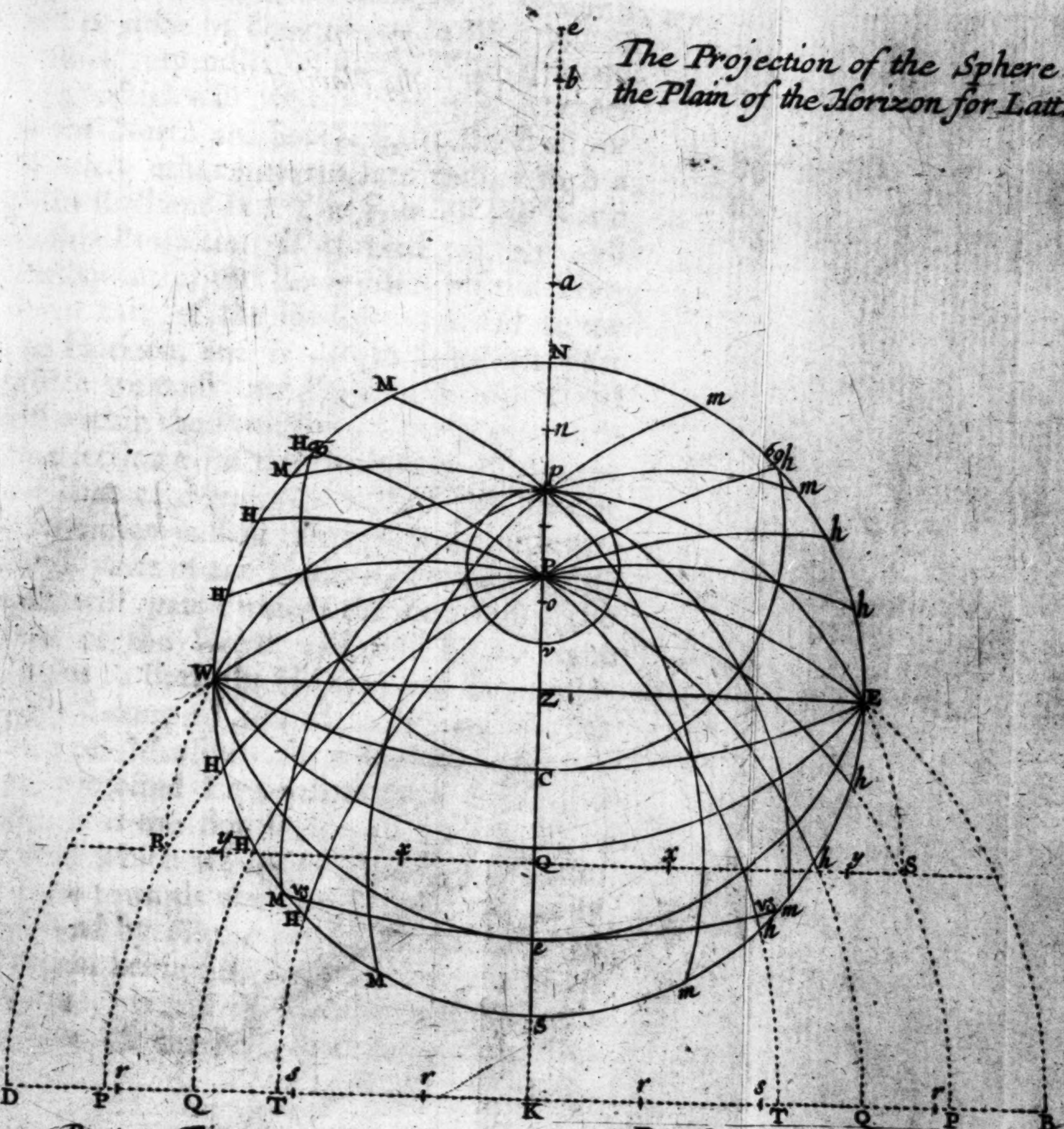
Draw a Circle of any Magnitude at Pleaſure, as N E S W repreſenting the Horizon ~~; in~~ which, draw the two Diameters W E and N S croſſing one another at right Angles, which will be the Repreſentations of two great Circles of the Sphere, as I ſhall ſhew afterwards: Let N repreſent the North, E the Eaſt, S the South, and W the Weſt part of the Horizon.

N. B. In all theſe Projections, we uſually ſuppoſe the Eye in the under Pole of the Primitive, Projecting that Hemiſphere which is oppoſed to the Eye, which will all fall within the Primitive; But that Hemiſphere in which the Eye is, will all fall without the Primitive, and will run out into an infinite annular Plain, in the Plain of the Projection, and conſequently cannot all of it be projected by Scale and Compaſs.

I. Let us begin with the Projection of the Equinoctial, and here we muſt firſt determin the Line of Meaſures in which the

Centre

The Projection of the Sphere upon the Plain of the Horizon for Latt. 51. 30'.



Part 2. Fig. 1.

P. 108.

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Centre of this Circle will be; and this will be done by determining in what points a plain perpendicular to the Primitive and Equinoctial will cut the Horizon, whether in the North and South, Eaſt and Weſt, or in what other intermediate points ſuch a plain ſhall cut it: The Pole of the World in this Projection is elevated 51°. 30′, and conſequently, the Equinoctial on the Northern part of the Horizon will fall below the Horizon, and it is the Southern part which we muſt here Project, or which will fall within the Primitive; That plain whoſe Interſection with the Horizon ſhall produce the Line of Meaſures, will be the plain of a Meridian paſſing through the North and South parts of the Horizon, for that Meridian will paſs through the Zenith and the Pole of the World, and will be at right Angles to both the Horizon and Equinoctial (*by Lemma* 2.) whereſore NS will be the Line of Meaſures; in which the Centre of the projected Equinoctial muſt fall; And ſince it is the Southern part of the Equinoctial which we are to project, its Centre will be towards the North, and may eaſily be found by *Prop.* 4. its Elevation above the Horizon being firſt known; and this in all Latitudes is ſtill the Complement of the Elevation of the Pole, ſo that the Poles Elevation in our Latitude being 51°. 30′, the Elevation of the Equinoctial above the Horizon will be 38°. 30′; the Tangent of which being ſet from *z* to *n* will give its Centre, and the Secant of the ſame 38°. 30′. will give its Radius (by *Cor. Prop.* 4.) with which the Circle W Q E may be deſcribed, which is the Repreſentation of that part of the Equinoctial, which is above our Horizon for Latitude 51°. 30′.

II. We will Project the Ecliptick, which being a great Circle of the Sphere, muſt cut the Equinoctial at a Diameter's Diſtance *i. e.* in E and W, the Eaſt and Weſt points of the Horizon; and conſequently will have the ſame Line of Meaſures with that of the Equinoctial, *viz.* N S, for one and the ſame Circle will paſs thro' the Zenith, the Pole of the Equinoctial and the Pole of the Ecliptick, and, by the ſecond *Lemma*, will be at right Angles to them all: The Line of Meaſures being determined, let us conſider whether its Centre will fall towards the North or towards the South of the Horizon; and this will eaſily be determin'd, by conſidering that the Equinoctial is elevated above the Southern Part of the Horizon 38°. 30′. and the Northern Part of the Ecliptick, or the Northern Signs, are elevated above the Equinoctial 23°. 30′. which, in all, make 62°, which is leſs than 90°. ſo that it muſt fall towards the South, and conſequently its Centre muſt fall towards the North, and may be found by *Prop.* 4. by ſetting the Tangent of 62°. from Z to *b*, and the Secant of 62°. will give its Radius, with which the Circle W C E, the Repreſentation of the Northern Half of the Ecliptick, may be deſcribed.

The Southern Part of the Ecliptick is likewiſe for the moſt part projected in the Horizontal Projection, and made to fall within the Primitive; but this cannot be the Globe remaining fixt, for that part of the Ecliptick which is below the Horizon will be thrown out of the Primitive; ſo that it cannot be projected unleſs the Globe be ſuppoſed to be turned round, and by that Means the Southern Part of the Ecliptick to be brought above the Horizon; but ſuch a Revolution of the Sphere, where it makes any Alteration, is ſcarce allowable; however I ſhall ſhow how it is uſually Projected.

The ſame Line of Meaſures NS remains ſtill, and the Circle muſt fall to the South, and conſequently its Centre to the North of the Horizon; therefore Nothing remains but to find its Elevation above the Horizon:

zon: The Northern Part of the Ecliptick falls 23°. 30′ nearer the Zenith than the Equinoctial does, therefore the Southern Part being brought above the Horizon, muſt be 23° 30′ nearer the Horizon than the Equinoctial; ſo that $38^\circ\ 30' - 23^\circ\ 30' = 15^\circ$ is the Diſtance of that part of the Ecliptick above the Horizon: It will be repreſented by W *e* E, and may be deſcribed by *Prop.* 4.

III. N S produced will likewiſe be the Line of Meaſures, for all parallels of Declination, and Parallels of Latitude for the Poles of leſſer Circles being the ſame with thoſe of the greater Circles to which they are parallel, 'tis certain, that the ſame Plain which is at right Angles to the Equinoctial and Horizon, will likewiſe be at right Angles to all leſſer Circles parallel to the Equinoctial; and the ſame will hold with regard to Circles parallel to rhe Ecliptick: But N S is the Line of Meaſures of the Equinoctial and Ecliptick, and conſequently muſt be the Line of Meaſures of all Circles parallel to either of them; therefore the Centers of ſuch leſſer Circles will be in N S produced if occaſion be; and they may be projected by *Prop.* 6 and 7: For Inſtance, the Tropick of *Cancer* may be deſcribed by *Prop.* 6. after this manner; conſider in this Poſition of the Sphere what will be its neareſt and greateſt Diſtance from the Zenith, or the Pole of the Primitive, which you will find to be 28°. for the Equinoctial being elevated 38°. 30′. above the Horizon, and the Tropick of *Cancer* being 23° 30′ from the Equinoctial, which, being added together, give 62°. which, ſubtracted from 90°. leaves 28°. its Diſtance from the Zenith on the South Side of the Horizon; therefore the half Tangent of 28°. or the Tangent of 14 ſet from Z to C, will give one Extremity of its projected Diameter. Then the Diſtance from the Zenith to the Pole being 38°. 30′. and from the Pole to the Tropick of *Cancer* 66°. 30′. the Sum of theſe, *viz.* 105°. will be its greateſt Diſtance from the Zenith, the half-Tangent of which Diſtance being ſet from Z to *a*, will give the other Extremity of its projected Diameter; therefore, having got C*a* the Diameter, 'tis but biſſecting it, and the Circle ♋ C ♋ may be deſcribed.

The Tropick of *Capricorn* will be deſcrib'd after the ſame Manner by *Prop.* 6. The Diſtance between the Equinoctial and the Zenith being 51° 30′. if to this be added 23°. 30′. you'll have 75°. equal to the neareſt Diſtance of the Tropick of *Capricorn* on the South Side of the Horizon ; the half-Tangent of which being ſet from Z to *e*, will give one Extremity of its Diameter. Then the Diſtance between the Zenith and the Pole, *viz.* 38°. 30′. and the Diſtance between the Pole and the Equinoctial, which is 90′. and the Diſtance between the Equinoctial and the Tropick of *Capricorn*, which is 23°. 30′. being all added together, will give the greateſt Diſtance of the Tropick of *Capricorn* from the Zenith, *viz.* 152. the Semi-tangent of which being ſet from Z towards the North, will give the other Extremity of the Diameter. Biſſect the Diameter found in *e*, and deſcribe the Circle ♑ *c* ♑, which is the Repreſentation of ſo much of the Tropick of *Capricorn* as falls within the Primitive.

IV. The Polar Circle may be deſcribed by *Prop.* 7 for it will fall on one Hand of the Zenith, as you will eaſily perceive by reaſoning after this Manner : The Diſtance of this Circle every where from the Pole is 23°. 30. but the Pole being Elevated on the North Side of the Horizon 51°. 33′. But 51°. 30′ + 23°. 30 = 75°. is leſs than 90°. ſo that it does not paſs beyond the Zenith. Therefore 75 ſubtracted from 90°.

leaves 15°. which is the neareſt Diſtance of the Polar Circle from the Zenith: And the half-Tangent of 15°. ſet from Z to *v*, will give one Extremity of its projected Diameter; and then 15°. + 47°. = 62°. will be its greateſt Diſtance from the Zenith the half-Tangent of which Diſtance ſet from Z to *p*, will give the other Extremity of its projected Diameter; ſo that its Diameter *v p* being found, 'tis but biſſecting it and the Circle may be deſcribed.

I ſhall omit the drawing any Parallels of Latitude leaſt I ſhould too much perplex the Scheme: But there will be no Difficulty in them, for they will be drawn after the ſame Manner as the Parallels of Declination are; for you muſt conſider their greateſt and neareſt Diſtance from the Pole of the Primitive, or the Zenith, and then ſet the half-Tangents of thoſe Diſtances found, either the ſame Way or contrary Ways from the Centre, according to the Circles to be projected fall on one Side or both Sides the Zenith, and then you will have their projected Diameters, and may deſcribe the Circles.

V. I ſhall now ſhow how to project the Hour-Circles; and firſt I am to determin a

Line of Meaſures, in which their Centers ſhall be if it be poſſible: But I eaſily diſcover it impoſſible to find one Line of Meaſures for them all; for they are differently enclined to the Horizon, and conſequently the Plain of no one great Circle can be at right Angles to the Horizon and all the Hour-Circles: Therefore let us ſee whether we can find the Plain of a great Circle at right Angles to the Horizon and any one of them; and this I find poſſible by conſidering that the Hour Circles being all at right Angles to the Equinoctial, their Poles will be all found in this Circle; but the Poles of all great Circles being 90°. diſtant from their Plains the Hour-Circle of 12 and the Hour-Circle of 6 muſt of Neceſſity paſs thro' each others Poles, and conſequently will be at right Angles to one another, by the ſecond *Lemma*. But the Hour-Circle of 12 is at right Angles to the Horizon from what was ſaid before, and interſects it in N S; therefore the Line N S will be the Line of Meaſures, in which will be the Centre of the Hour-Circle of 6, and its Centre will be towards the South Part of the Horizon, becauſe all the Hour-Circles paſs thro' the Pole which falls towards the North. The Elevation of thi Circle above the Horizon being the ſam

wit

with that of the Pole, *viz.* 51°. 30′. take the Tangent of 51°. 30′. and ſet it from Z to K; and upon the Centre K, and with the Secant of the ſame Elevation deſcribe W P E, which is the Circle requir'd.

The Point P where N S and W E interſect one another, is the Repreſentation of the Pole of the World; for N S being the Repreſentation of the Hour-Circle of 12, the projected Pole muſt be ſomewhere in this Line; but it muſt be ſomewhere in W E, which is likewiſe the Projection of an Hour-Circle, therefore it muſt be in that Point where theſe two projected Circles interſect one another, that is, in the point P; P is the point through which all the Hour-Circles muſt paſs in the Projection.

In order to draw the reſt of the Hour-Circles we muſt have Recourſe to a *ſecondary* Line of Meaſures, which will be determined after this Manner: To P K at the point K, erect D B at right Angles, and produce the Circle W P E till it meet the Line D B in the points D and B, I ſay, the Line D B will be the ſecundary Line of Meaſures in which the Centers of all the Hour-Circles will be found; for let the Hour-Circle of 6 D P B be conſidered as the primitive Circle, in whoſe under

Pole

Pole (which will be in the Equinoctial) K let the Eye be placed; then will D B be the Repreſentation of the Equinoctial, for it paſſing thro' the Eye will be projected into a right Line; but the Equinoctial is at right Angles to the Hour Circles, both the Primitive and all the reſt, therefore it will be the *ſecondary* Line of Meaſures upon this Suppoſition, in which will be all their Centers: In order to the finding of which ſet the Sector to the Radius P K, then take of the Tangents of 15°. 30°. 45°. the Elevations of the Hour-Circles above the Hour-Circles at 6, and ſet them both Ways from K to *r*, from K to *ſ*, and from K to *r*, &c. then upon thoſe Centers and with the Secants of the ſame Elevations, deſcribe the Circles P P, P Q and P T, which will be the Hour-Circles; for they are all great Circles of the Sphere, (for the Suppoſition does not make any Alteration as to that) paſs thro' the pole P, and make Angles with one another of 15°. or are 15°. diſtant from each other, and the Portions of thoſe Circles which fall within the Primitive N E S W as H P *h* are the Repreſentations of thoſe Halves of the Hour Circles which are above our Horizon in our Latitude,

VI. In like Manner, the Circles of Longitude may be drawn, by determining the Secundary Line of Meaſures RS in which all their Centers will be ; and this Line will be determined after the ſame Manner with DB above, and the Circles of Longitude drawn as we there drew the Meridians: For the Line NS will be the Line of Meaſures, with reſpect to one of them paſſing through E and W the Eaſt and Weſt Points of the Horizon ; in order to draw this Circle conſider how much it is elevated above the Horizon, and this will be found by conſidering the Diſtance of the Pole of the Ecliptick from the Pole of the World, which is 23°, 30′. but the Elevation of the Pole is 51°. 30′. therefore 51°. 30′. − 23°. 30′. = 28′. will be the Elevation of this Circle above the Horizon ; ſet the Tangent of 28°. from Z to Q, and with the Secant of the ſame Diſtance deſcribe the Circle W*p*E ; to *p* Q at the point Q erect R S at right Angles, which will be the Secundary Line of Meaſures ; in this Line from Q, (the Sector being ſet to *p* Q) ſet off the Tangents of 20°. 40°. according to the Number of Circles you have a Mind to draw, from Q to *x*, from Q to *y*, &c. and with the Secants of 20°. 40°. &c. de-

deſcribe the Circles of Longitude M *p m*, &c.

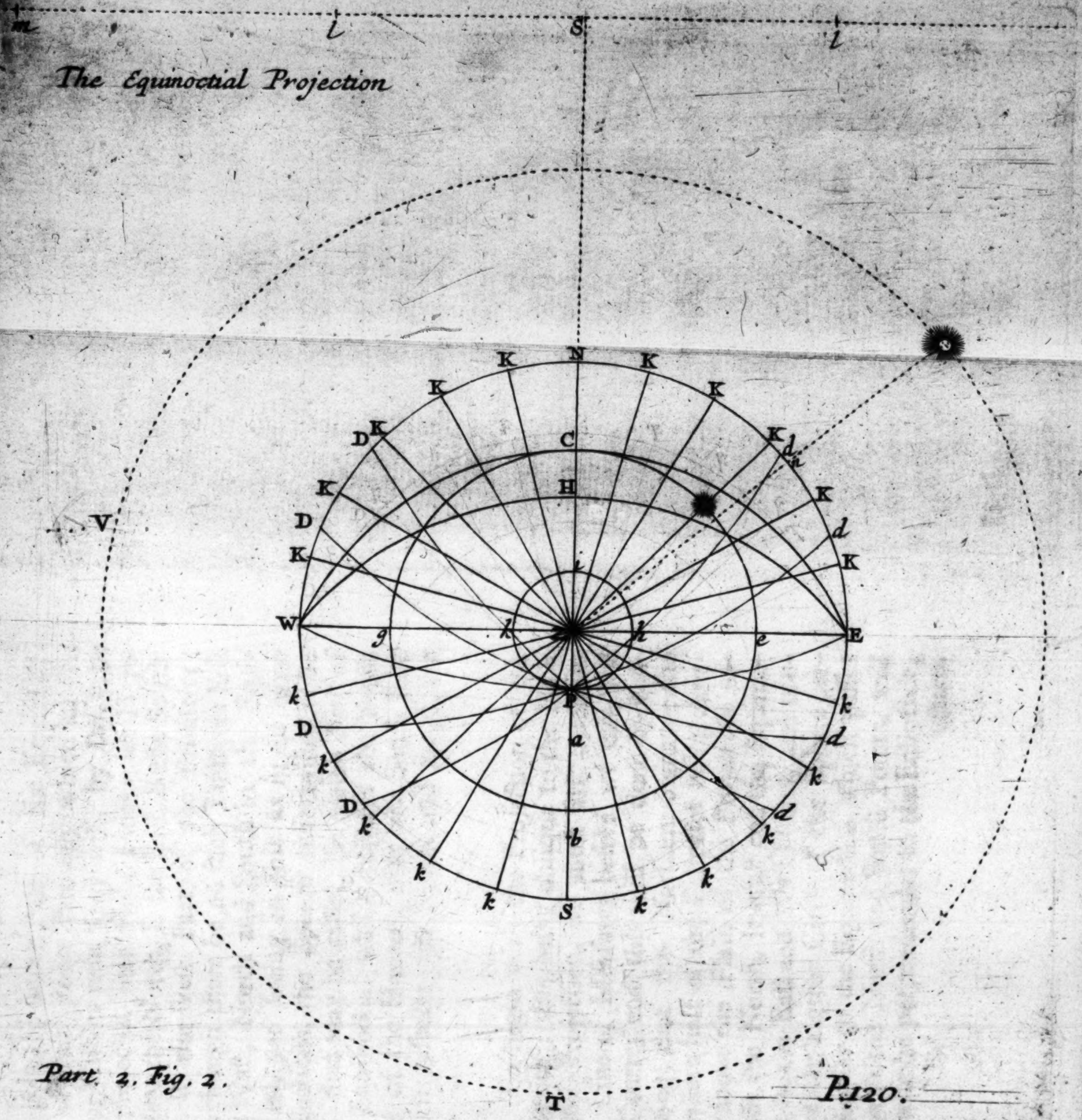
VII. The Repreſentation of Azimuths in this Projection will be all right Lines; and any Number of them may be drawn making any aſſigned Angles with one another, if the Limb be divided into its Degrees by the help of a Sector, and through theſe Degrees be drawn Diameters to the Primitive.

VIII. All Parallels of Altitude in this Projection will be Circles parallel to the Primitive, as may be gathered from the 6 *Prop.* and may be eaſily drawn by that *Prop.* ſo that I ſhall omit the drawing of any of them, for fear of too much perplexing the Scheme.

The Projection of the Sphere upon the Plain of the Equinoctial.

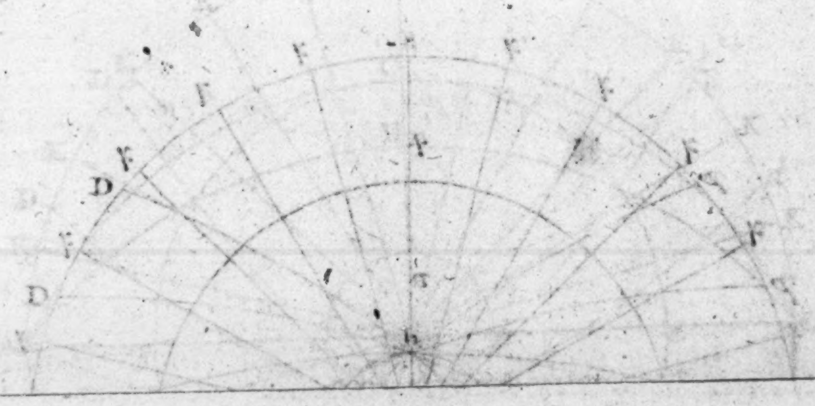
Deſcribe the Circle W N E S repreſenting the Equinoctial or the Primitive Circle, and draw the Diameters W E and N S croſſing one another at right Angles, which will be the Repreſentations of two Hour-Circles which in this Projection will be all projected into right Lines; for the Eye being

The Equinoctial Projection



Part. 2. Fig. 2.

P.120.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being in the under Pole of the Equinoctial, or that Point through which all the Hour Circles paſs, they will, by *Def.* 5. be all projected into right Lines; let N repreſent the North, E the Eaſt, S the South, and W the Weſt Part of the Primitive; and though there be no ſuch Thing as Eaſt and Weſt, North and South in the Equinoctial, yet ſeeing in this as in all other Projections, the Sphere is imagined to be fixt, that may be called the North Part of the Equinoctial which reſpects the North Part of the Horizon, that the South Part which reſpects the South, and ſo of the reſt.

I. To begin with the Projection of the Ecliptick: this being oblique to the Plain of the Projection, we muſt firſt determin the Line of Meaſures before we can find its Centre, and this will be done by conſidering, that the Northern Signs or the Northern half of the Ecliptick being elevated above the Plain of the Projection towards the North, it muſt cut the Equinoctial in the Eaſt and Weſt Points, and conſequently a great Circle of the Sphere perpendicular to the Equinoctial which paſſes through the North and South Points, will likewiſe be perpendicular to the Ecliptick;

wherefore its Line of Meaſures will be NS, and its Centre may be found by ſetting the Tangent of 23° 30′. its Elevation of above the Primitive from Z to *a*, and with the Secant of the ſame Elevation the Circle W C E may be drawn, which is the Repreſentation of the Ecliptick.

II. The Northern Part of the Horizon for our Latitude 51°. 30. will likewiſe be elevated above the Equinoctial, wherefore it will interſect the Equinoctial in the Eaſt and Weſt Points, and conſequently will have the ſame Line of Meaſures with the Ecliptick, *viz.* N S: Its elevation above the Primitive is 38°. 30′. wherefore take the Tangent of 38°. 30′. and ſet it from Z to *b*, which will give its Centre, and with the Secant of the ſame Elevation deſcribe the Circle W H E, which is the Repreſentation of our Horizon.

III. All Parallels of Latitude and Almacanters, or Parallels to the Ecliptick and Horizon, will have the ſame Line of Meaſures with their reſpective greater Circles, *viz.* the Ecliptick and Horizon to which they are parallel; *i. e.* N S will be the Line of Meaſures for all ſuch Circles; and they may be eaſily drawn by *Prop.* 6 and 7. after the ſame Manner that the Parallels of Declination were drawn in the Horizontal Projection.

IV. The Parallels of Declination will be Concentrical with the Primitive; ſo that there will be no Occaſion to determin a Line of Meaſures for theſe Circles; for they may be drawn by *Prop.* 6. by taking the Semi-tangents of their reſpective Diſtances from the Pole, and placing one Foot of the Compaſſes in the Centre of the Primitive, and therewith deſcribing the Circles: Thus the Tropick of *Cancer* C *f g* may be drawn by taking the half-Tangent of 60°. 30. and therewith placing one Foot of the Compaſſes in the Centre deſcribing a Circle, which will be the Tropick required.

V. The Hour-Circles in this Projection will be all repreſented by right Lines; and in order to draw theſe, you need only ſet off the Chords of 15°. 30′. 45°. 60°. 75°. and 90°. from W and E towards N, and then draw the Diameters K Z *k* and 'tis done; for thoſe Diameters will be the Repreſentations of the Hour-Circles.

VI. Circle of Longitude and Azimuths will here be drawn after the ſame way; I ſhall only ſhow the way to deſcribe, 3 or 4 of the Circles of Longitude for by this any one will eaſily perceive how to draw the Azimuths: And firſt, the Line NS being the common Interſection of a Circle perpendicular to the Equinoctial and Ecliptick, with the Plains of thoſe two Circles, the Repreſentations of both their Poles muſt be ſomewhere in the Line NS; but the Ecliptick being elevated above the Equinoctial towards the North, its Pole muſt of courſe be towards the South; but its Pole is ſomewhere in the Circle *phik*, which repreſents the polar Circle, therefore it is in the point *p*, where *phik* and NS interſect one another towards the South; *p* is the Point where the Circles of Longitude interſect one another: Find the Centre of a Circle paſſing through the 3 Points W, P and E, which will be eaſily done from the Elements, which let be S; 'tis plain that W PE will be the Repreſentation of a great Circle of the Sphere, as cutting the Primitive in W and E, the ſame Points that the Ecliptick does, and conſequently will be at right Angles to the Ecliptick by *Lemma* 2. and by the ſame *Lemma* it is evident, that a great Circle which paſſes thro' the Poles of one, and is perpendicular to it, will likewiſe paſs through the Poles of the other and be perpendicular to it; wherefore N S will be the Line of Meaſures for the Circle W P E, and being produced will paſs thro' its Centre S; erect *m m* at right Angles to NS, and in this Line from the point S, ſet off the Tangents of 20°. 40°. *&c.* according to the Number and Diſtances of the Circles you have a Mind to draw, and then, according to *Prop.* 4. with the Secants of 20°. 40°. deſcribe the Circles D*d*, &c. The Reaſon of proceeding thus in determining the Line *m m* is evident from *n* 5 in the Horizontal Projection.

VII. If it were required to project the fixed Stars in this Projection, find the Declination and right Aſcenſion of the Star to be Projected, which may be had from the Aſtronomical Tables, and then proceed thus: Suppoſe the Declination of a Star were 23°. 30′. North, and his right Aſcenſion 45 take the Semi-tangent of the Complement of 23°. 30′. *viz.* 66°. 30′. and therewith placing one Foot of the Compaſſes in the Centre of the Primitive, deſcribe the Circle C *e g*, which will be the Star's Parallel of Declination; then ſet off 45°. its right Aſcenſion, from E to K, and draw K Z, and that Point where K Z interſects the Parellel will be the Star's place.

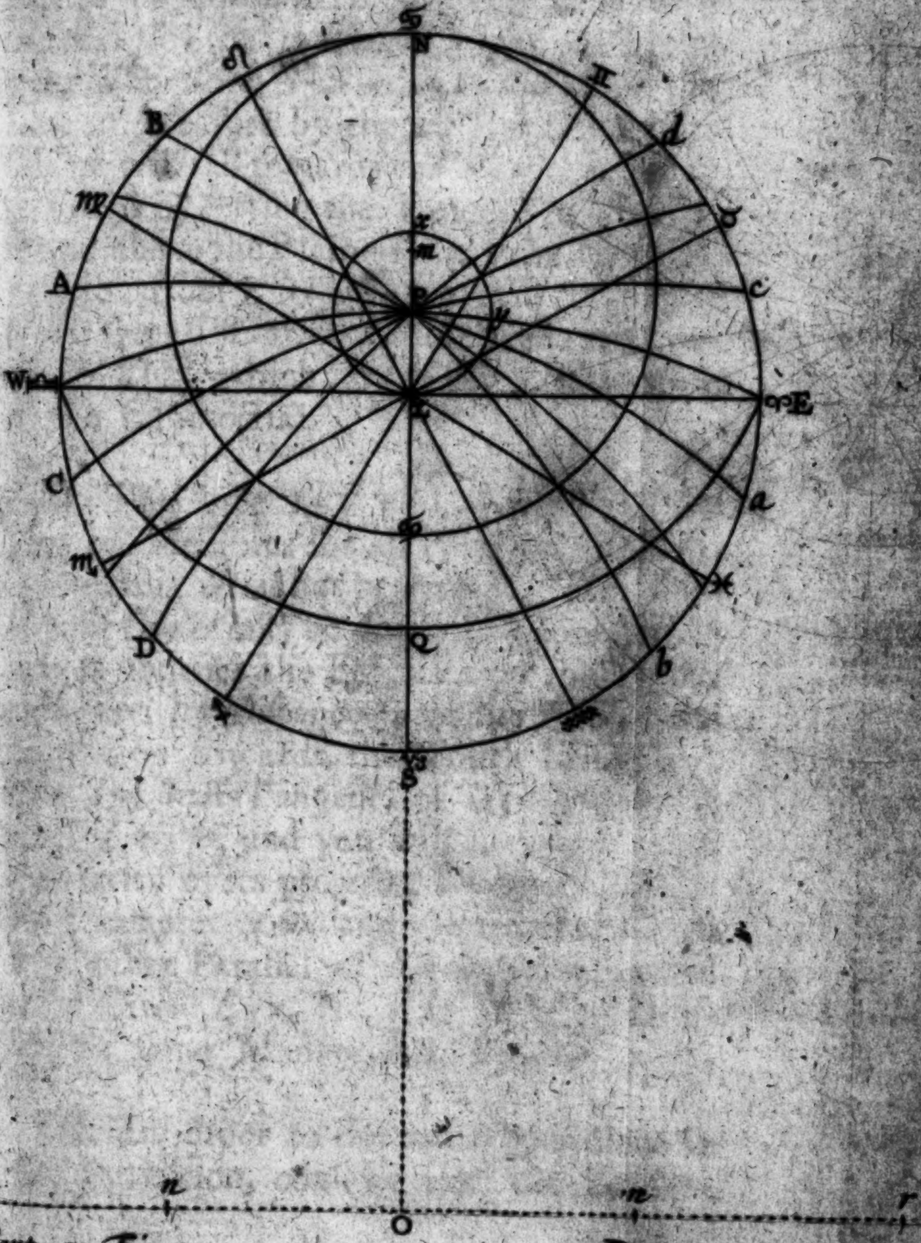
But if the Declination of the Star had been 30°. South, and its right Aſcenſion 40°. then take the Semi-tangent of 90°. + 30°. = 120°. and therewith deſcribe the prickt Circle * T V; which will be the Star's Parellel of Declination, as will be evident by the Stereographick Problems, and then ſet off the Chord of 40°. from E to *n*, and draw the prickt Line Z *n* produced till it meet the Stars Parallel of Declination, and that Point in which it meets it will be the Stars place.

The Projection of the Sphere upon the Plain of the Eclip tick.

Deſcribe the Circle N E S W, repreſenting the primitive Circle or Plain of the Ecliptick, and croſs it with the two Diameters N S and W E, which will be the Repreſentations of two Circles of Longitude, all which in this Projection will be repreſented by right Lines, ſince they all paſs thro' the Eye which is placed in the under Pole of the Ecliptick: Let N repreſent the North, E the Eaſt, S the South, and W the Weſt Part of the Ecliptick.

I. The Equinoctial will here fall towards the South, for 'tis the Northern half of the Ecliptick that is elevated above the Equinoctial,

Stereographick Projection of the Sphere upon the
n of the Ecliptick.



Part 2, Fig. 3.

P. 126.

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noctial, and conſequently, 'tis the Southern part of the Equinoctial (if I may be allow'd to ſpeak ſo) that is elevated above the Ecliptick: Whereſore, NS being found to be the Line of Meaſure by the laſt Projection, its Centre will fall towards the North, and may be found by ſetting the Tangent of its Elevation above the Plain of the Primitive, *viz.* 23°. 30′. from Z to *m*; then take the Secant of the ſame Elevation and deſcribe the Circle W Q E, which is the Repreſentation of the Equinoctial.

II. The Line N S will likewiſe be the Line of Meaſures for all Parallels of Declination from what has been ſaid. The Tropick of *Cancer* may be drawn, by conſidering that it touches the Ecliptick on the North Side, and its Diſtance from the Pole of the Projection towards the South being 43°. take the half-Tangent, of 43°. and ſet it from Z to ♋, and you will have the other Extremity of its projected Diameter; biſect the Diameter ♋ ♋, and deſcribe the Circle Any other Parallel of Declination, as the polar Circle Z *y x*, &c. may be deſcibed after the ſame Manner, *viz.* by *Prop.* 7.

III. In order to draw the Meridians in this Projection, conſider that the Equinoctial being elevated above the Plain of the Projection towards the South, its Pole muſt fall ſomewhere in the Line NS towards the North, and may be found by *Prop.* 1. by ſetting off the half-Tangent of 23°. 30′. from Z to P. Through P and the Eaſt and Weſt Points E and W, draw the Circle W P E, which will be the Repreſentation of the hour-Circle of *6*; at *o* the Centre of this Circle, erect *r r* at right Angles to PO; and this will be the *Secondary* Line of Meaſures, in which Line all their Centers will be and may be found, by ſetting off the Tangents of 15°. 30′. *&c.* both ways from *o* to *n*, from *o* to *r*, &c. and the Circles AP*a*, BP*b*, CP*c*, DP may be deſcribed with the Secants of 15°. 30°. *&c.* as above.

IV. The Line NS will likewiſe be the *Line of Meaſures for our Horizon,* and likewiſe all Parallels of Altitude; the firſt of which may be drawn by *Prop.* 4. and the latter by *Prop.* 6. and 7.

V. The Azimuth may be drawn by finding the Pole of the Horizon in the Lin NS towards the North, and determinin a *Secondary* Line of Meaſures as above.

VI. Th

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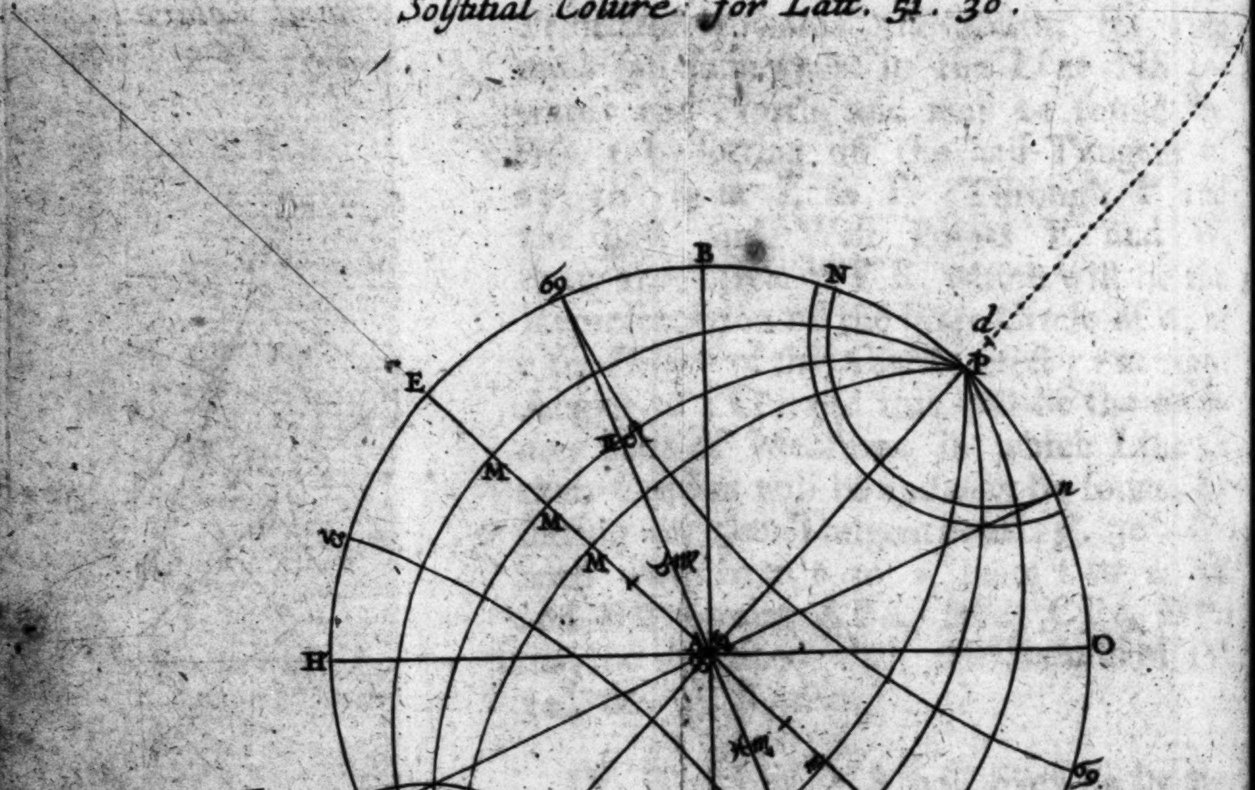
The Projection of the Sphere upon the Plain
Solſtitial Colure for Latt. 51. 30′.



Part 2. Fig. 4.

P. 129.

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VI. The Stars may be repreſented in this Projection, by knowing their Longitude and Latitude, after the ſame Manner that they were in the Equinoctial Projection, having given their Right *Aſcenſion* and *Declination.*

The Projection of the Sphere upon the Plain of the Solſtitial Colure. The Pole being elevated 51° 30′. *above the Horizon,*

Draw the Circle **HBOC**, repreſenting the Primitive Circle or Solſtitial Colure, and the Diameter **HO** repreſenting the Horizon; ſet off the Chord of 51°. 30′. from O to P, which will give the polar Point, nd draw the Diameter P*p*, repreſenting he Hour-Circle of 6.

I. The Epuinoctial may be repreſented y drawing the Diameter EQ at right ngles to the Diameter P*p*.

II. Set off 23°. 30′. from the Chords rom E to ♋ and from Q to ♑ and draw he Diameter ♋ ♑, which will repreſent he Ecliptick.

III. The Tropicks of *Cancer* and *Capricorn* may be drawn thus; each of theſe Circles being 23°. 30′. diſtant from the Equinoctial, muſt be *66*°. diſtant from their reſpective Poles; therefore take the Secant of *66*° 33′. and ſet it both Ways from the Centre A in P*p* produced, (for P*p* will be the Line of Meaſures for all Parallels of Declination) which will give the two Points *e e*, the Centers of the two Circles, by *Prop.* 5. and their Radii will be the Tangents of the ſame *66*°. 30′ by the ſame Propoſition.

IV. The Polar Circles, as likewiſe all other Parallels of Declination, may be drawn after the ſame Way by *Prop.* 5.

V. The Line of Meaſures for the Azimuths will be H O, and they may be drawn by *Prop.* 4. And the Line of Meaſures for the Almacanters will be BC, and they may be drawn by *Prop.* 5.

VI. ♋ ♑ or the Ecliptick will be the Line of Meaſures for the Circles of Longitude; and they may be drawn by *Prop.* 4. And the Line of Meaſures for the Circles of Latitude will be *n* S, and they may be drawn by *Prop.* 5.

VII. *N. B.* The Ecliptick may be divided into its proper Signs in this Projection, by ſetting off the Tangents of 15°. 30°. 45°. from A both Ways to ♉ and ♓, from A to ♊ and ♒, *&c.* and to thoſe Diviſions placing their proper Signs, beginning at the Centre, where muſt be placed ♈ and ♎, for the Eye being at the Interſection of the Equinoctial and Ecliptick, the two Points of *Aries* and *Libra* will be projected into the Centre.

Theſe are the Projections which are of greateſt Uſe, and though there may be a great many more upon the Plains of other great Circles, yet if the Reader thorowly underſtand theſe, he cannot be at a loſs how to project the Sphere upon the Plain of any great Circle, which he ſhall ever have Occaſion for; now I ſhall proceed to the Stereographick Problems, in which I ſhall ſhow how to meaſure any Ark or any Angle of any Circle projected, *&c.*

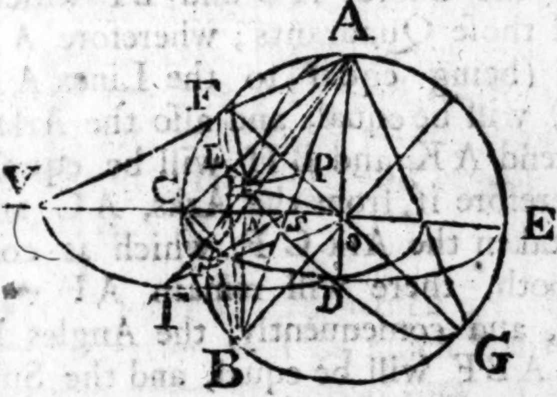
Stereographick Problems.

The Degrees of any oblique Circle upon the Sphere, are all equal to one another; but the Repreſentations of thoſe Degrees in the Plain of the Projection, are very unequal, as is evident from *Fig. pag.* 98. A *e* G *f* is the Repreſentation of an oblique Circle upon the Sphere, one half of which is projected into A *e* G and the other into G *f* A; but theſe two are very unequal to one another, nor are the Degrees in each of theſe equal amongſt themſelves, ſo that in the firſt Place I ſhall ſhow how to divide the Repreſentation of any oblique Circle into its proper Degrees; in order to do which I premiſe this *Lemma.*

LEMMA III.

If thro' the remoteſt Poles of two equal Circles in the Sphere, any other Circle be drawn, it will cut off equal Portions from the Peripheries of the two equal Circles given.

Thoſe Circles in the Sphere which are equal are either great Circles, (all which are equal) or leſſer ones, and one and the ſame Demonſtration wou'd ſerve for all; but



but to avoid Confuſion, I ſhall firſt Demonſtrate the *Lemma* true of great Circles, and then a-part ſhow the truth of it in leſſer ones.

I. Let A be the upper Pole of the great Circle C D E, and B the lower Pole of the great Circle F D G, and through theſe two remoteſt Poles of the two great Circles, the upper A, and the under B, let any Circle be drawn as A H K B; I ſay that this Circle cuts off equal Portions of the Circles C D E and F D G, *i. e.* C K is equal to F H, and D H equal to D K, ſince A is the Pole of the Circle C D E, the Arks A C and A K will be equal to one another by the Definition of a Pole. And the Arks A F C and F C B are alſo equal, being

Quadrants of the ſame Circle, as are likewiſe the Chords A C and B F which ſubtend thoſe Quadrants; wherefore A K and B H (being equal to the Lines A C and B F) will be equal, and alſo the Arks they ſubtend A K and B H will be equal too; Therefore if from the Arks, A C and B F be taken the Ark C F, which is common to both, there will remain A F equal to B C, and conſequently, the Angles B A C and A B F will be equal, and the Subtenſe L A equal to the Subtenſe L B. In like Manner, if from the equal Arks AHK and BKH, be taken the Ark K H common to both, there will remain the Ark A H equal to the Ark B K; wherefore the Angles A B H and B A K, and likewiſe the Subtenſes of thoſe Angels A N and N B will be ſeverally equal; Therefore in the Triangles L A N and L B N, ſince the Sides A L and A N are equal to the Sides B L and B N, and the Side L N common to both, the Triangles will be ſimilar, and the Angle L A N equal to the Angle L B N. Again, in the Triangles C A K and F B H, bcauſe the Sides A C and A K are equal to the Sides B F and B H, and the Angle C A K equal to the Angle F B H, (as was proved from the Similarity of the Triangles L A N and L B N) the Triangles will be equal, and the Side C K equal to the Side FH. But in equal Circles equal Chords Subtend equal Arks, therefore the Ark CK is equal to the Ark F H, and if from the equal Arks DC and DF be taken, the equal Arks C K and F H, there will remain DH equal to DK. Q. E. D.

II. As to leſſer Circles which are equal, the Demonſtration will be much the ſame. For becauſe they are equal they will be equally Diſtant from the Centre of the Sphere, and conſequently equally Diſtant from their remoteſt Poles, for the Centre of a great Circle is the middle Point between the Poles of that Circle. Wherefore all the right Lines which can be drawn from the Poles of thoſe Circles to their Peripheries will be equal, as in the *1ſt* Part A C and A K were equal to B F and BH, and wou'd be ſtill ſo, though the Circles C D E and F D G were leſſer Circles, in caſe they were but equal : And ſince the Angle C A K in leſſer Circles as well as greater is equal to F B H the Baſes C K and FH will be equal here alſo ; and conſequently, the Arks C K and FH which they Subtend, will be equal alſo ; which was to be ſhown.

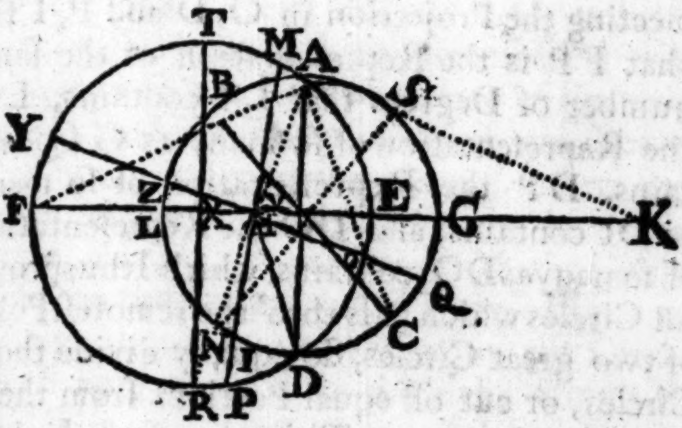
COROL.

From whence it is evident, That if AF and AH be produced until they meet the Plain, in which the Circle CDE and VTD the Projection of the Circle FDG are deſcribed in the Points V and T, SK produced will likewiſe fall upon T; for ſince AH and SK are in the ſame Plain, *viz.* the plain of the Circle AHKB, and are not parallel to one another, they muſt, if produced, of neceſſity meet. But SK is in the ſame Plain with T and V, and the Projection of the oblique Circle FDG; for I ſuppoſe T and V to be in the plain the Circle CDE; therefore SK being produced, will meet AH there where AH meets the Plain of the Circle CDE, that is, in T.

PROB. I.

To divide the Repreſentation of a great Circle into its proper Degrees.

Let ABDC be the Primitive, and LG the Line of Meaſures; let BC be the Diameter of an oblique Circle, produced by the Interſection of a Plain at right Angles to the Primitive, and the oblique Circle, with the ſaid oblique Circle, as L G is the Interſection of the ſame Plain with the Primitive; let the Eye be placed at A, projecting the Diameter B C, whoſe Repreſentations will be EF, and conſequently the Repreſentation of the oblique Circle B C will be EDF; N and S the Poles of the oblique Circle B C, will be projected into the Points H and K: This being premiſed, if from H the projected Pole of the oblique Circle B C, be drawn any number of right Lines to the Diviſions of the Primitive, and produced if Occaſion be until they meet the oblique Projection DF; I ſay, they will divide the Projection into its proper Degrees: For inſtance, if the Simi-circle G D L be divided into Q D and I, and thro' the



the Points Q D and I, and from the point H be drawn the Lines H Q, H D and HI, meeting the Projection in O, D and P, I ſay that F P is the Repreſentation of the ſame number of Degrees that L I contains, E O the Repreſentation of ſo many as G Q contains, D P the Repreſentation of ſo many as DI contains, and DO the Repreſentation of ſo many as DQ contains; which I thus prove; all Circles which paſs thro' the remoteſt Poles of two great Circles, do equally divide thoſe Circles, or cut off equal Portions from them both, by *Lemma* 3. This therefore I thus prove to be true here; B C and L G are the Diameters of two great Circles, one of which, *viz.* L G is directly oppoſed, and the other B C obliquely oppoſed to the Eye at A; A is the upper Pole of L C, and N the under Pole of B C; therefore all Circles of the Sphere which paſs thro' A and N, do cut of equal Portions from the Circles B C and L G: If therefore all theſe Circles be extended into one and the ſame Plain, or, which is all one, projected into the ſame Plain, the ſame thing will happen in the Projections, which happened in the Circles before they were projected; for the Circle L G will be produced into the Circle A L D G, and the oblique Circles B C will become the Circles E D F, and all thoſe Circles which paſs thro' the remoteſt Poles A and N, and cut off equal Portions from the two Circles BC and L G, will be projected into right Lines interſecting one another in H the Repreſentation of the two Poles A and N: Therefore what was ſhown Orthographically in the *Lemma*, is here repreſented Stereographically; for here the Circle A L D G is the ſame with the Circle CDE in the *Lemma*, being there oblique to the Eye, but here direct. Again, the Circle EDF here is the ſame with the Circle R DTV in the *Lemma*, both of them projected from an oblique Poſition of the Eye. Laſtly, H here is the Projected Pole of the oblique Circle BC, as there S is the projected Pole of the Circle F D G; and here the Line HIP, as in the *Lemma*, SKT repreſents the Projection of any one of thoſe Circles which paſs thro' the remoteſt Poles of the Circles propoſed: Wherefore, as in the *Lemma*, S T cutting the Circle CDE in K, does in like Manner cut RDV in T; ſo here HP cuts the Circle ALDG in I, and the oblique Projection EDF in P; therefore, ſince in the *Lemma* the Ark C K is equal to the Ark FH, and VT is the Projection FH, it is evident, that the Projection DVT is ſo cut in T as DHF is in H; ſo that here if H be the Pole of the oblique Circle, 'tis manifeſt, that right Lines drawn from H thro' any number of Degrees of the Circle A L D G, will being produced if occaſion be cut off in the Projection the Repreſentation of an equal number of Degrees; which was to be ſhown.

COR.

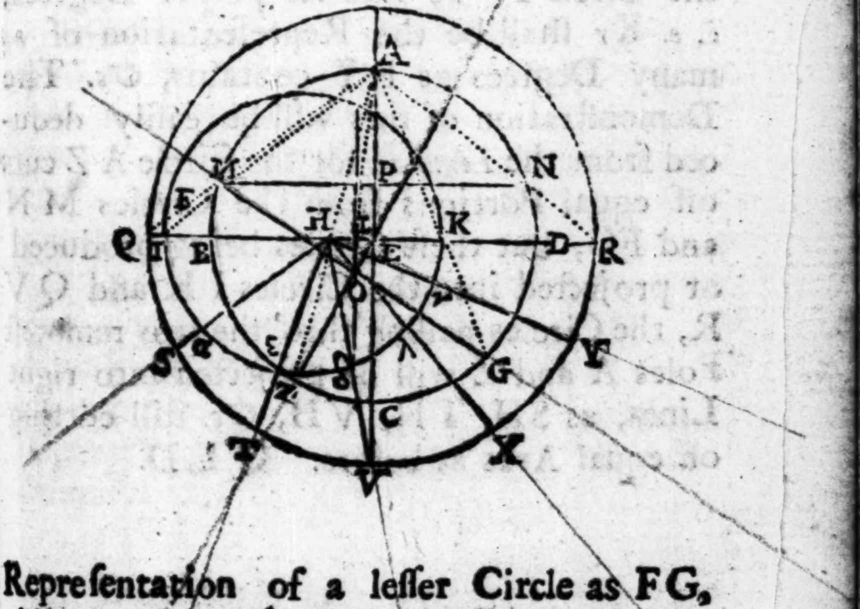
Hence we have an eaſie way by the help of a Sector, of dividing the Repreſentation of any oblique Circle into its proper Degrees; ſuppoſe it were requir'd to find the number of Degrees which O D repreſented (*See Fig. Prob.* 1.) firſt find H the projected Internal Pole of the oblique Circle, which I ſhall ſhow how to do in *Prob.* 3. and then from H, through the two Points D and O, draw the two Lines H D and H O produced until it meet the Primitive in Q; apply D Q to the Chords, and it will give the Number of Degrees which D O repreſents; and L I apply'd to the Chords ſet to the Radius of the Primitive, will give the Number of Degrees which F P repreſents, *&c.*

PROB.

PROB. II.

To divide the Repreſentation of a leſſer oblique Circle into its proper Degrees.

Det ABCD repreſent the Primitive whoſe Centre is E, and let I K be the



Repreſentation of a leſſer Circle as FG, oblique with reſpect to the Eye at A; its true Pole Z is projected into the Line of Meaſure BED at H, and its true Centre O projected into the Line of Meaſures at L; take EP equal to EO, and draw MN parallel to BD, which will be as far diſtant from the Centre E as F G is; then project the Diameter M N into Q E R; then with E Q or E R, and from the Centre E, deſcribe the Circle Q V R; divide the Circle laſt deſcribed into any number of equal Parts, ſuppoſe in the Points Y, X, V, T, S; and from the Pole H to the Points Y, X, T, V, S draw the Lines H S, H T, H V, H X, H Y, which Lines will divide the Circle I δ K into its proper Degrees, *i. e.* K*r* ſhall be the Repreſentation of as many Degrees as R Y contains, *&c.* The Demonſtration of this will be eaſily deduced from the *Lemma*; for the Circle A Z cuts off equal Portions from the Circles M N and F G, but theſe Circles being produced or projected into the Circles I K and Q V R, the Circles paſſing thro' the two remoteſt Poles A and Z will be projected into right Lines, as S H, T H, V H, *&c.* ſtill cutting off equal Arks as before. Q.E.D.

COROL.

COROL.

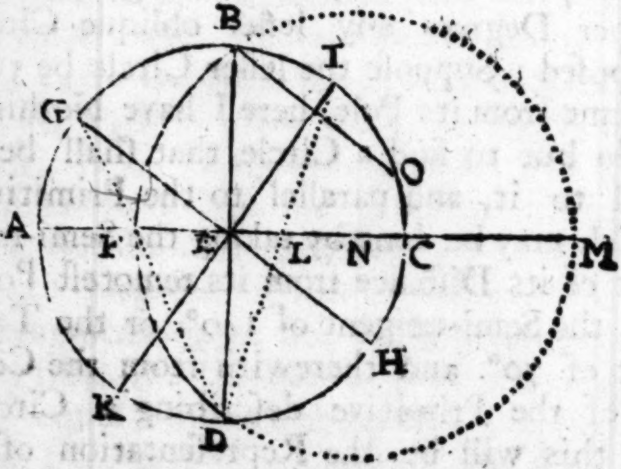
Hence we have a practical Method, by the help of a Sector, of dividing into its proper Degrees any leſſer oblique Circle propoſed; Suppoſe the leſſer Circle be 50°. Diſtant from its Pole, here I have Nothing to do but to find a Circle that ſhall be equal to it, and parallel to the Primitive, which may be done by taking the Semi-tangent of its Diſtance from its remoteſt Pole, *viz.* the Semi-tangent of 140°. or the Tangent of 70°. and therewith from the Centre of the Primitive deſcribing a Circle, for this will be the Repreſentation of a Circle equal to the Circle to be divided; and conſequently Lines drawn from the Pole H (which I ſhall ſhow how to find in the next *Problem*) to any Number of Degrees in the Circle Q V R, will in like Manner divide the Circle I K.

PROB III.

To find the Pole of any Circle in the Projection.

Here will be ſix Caſes; three with reſpect to the greater Circles, and three with reſpect to the leſſer ones; and

I. The Pole of a great Circle direct to the Eye will be its Centre; for the Eye



if it be direct, muſt be in its *Axis*, and conſequently its two Poles will be projected into the Centre.

II. The Pole of a right Circle may be found by ſetting of 90°. from one of its extream Points, thus; ſuppoſe the right Circle be A C, 90°. *(See laſt Fig.)* taken from the Chords and ſet from A to B (the Sector being fitted to the Circle ABCD) will give B its Pole; for A C being a right Circle, the Circle ABCD will be perpendicular to it; and conſequently (*by Lemma* 2.) will paſs through its Poles, and from the Definition of a Pole, B and D will be the Poles required.

III. The Pole of an oblique Circle may be found after this Manner; Let the oblique Circle be D F B *(See the laſt Fig.)* whoſe Pole it is required to find; imagin the Eye to be placed at D, and then thro' F that Point where D F B cuts the Line of Meaſures A C, draw the Line D G, and then thro' G and the Centre E, draw the Line G E H, which will be a Diameter of that oblique Circle upon the Sphere, of which D F B M is the Projection, and will likewiſe be that Diameter in which the Plain of a great Circle, as A B C D, at right Angles to the primitive and oblique Circle, cuts the oblique Circle; wherefore the Circle A B C D will paſs thro' its Poles, which may be found by ſetting 90°. from G to I and from G to K, for the two Points I and K will be its Poles, the firſt of which will be found in the Line of Meaſures A C at L, and the Projection of the other Pole K, will be that Point where a Line drawn through D and K meets A C produced.

The three other Caſes concerning leſſer Circles, are the ſame with the three former Caſes; for

I. The projected Pole of a leſſer Circle direct to the Eye, is the ſame with that of a greater Circle direct to the Eye, *viz.* its Centre.

II. The Pole of a leſſer Circle that is right to the Eye, is the ſame with the Pole of a great Circle to which it is Parallel, and conſequently, will be projected into the ſame Point: Or it may be better done thus, ſuppoſe B O (*Fig. pag.* 103.) a leſſer Circle, direct to the Eye at B, erect KE I at right Angles to the Diameter B O, and the two Points K and I will be its two Poles, and then join B K, and produce BI, and the two Points where thoſe two Lines meet the Line of Meaſures A C, will be the two projected Poles.

III. The projected Pole of a leſſer oblique Circle, is the ſame with that of greater oblique Circle to which it is Parallel.

PROB. IV.

To deſcribe a Spherical Angle of any Number of Degrees.

Before I proceed to the Solution of this *Prob.* it will be neceſſary to premiſe this Definition.

II. A *Spherical Angle* is the Inclination of two great Circles upon the Sphere meeting in a Point, and whoſe Meaſure is the Ark of a great Circle, intercepted between the two Arks which conſtitute the Angle at 90°. diſtance from the Angular Point.

There will be three Caſes of this *Prob.* for the Angular Point may either be at the Centre of the Primitive, at the Periphery of the Primitive, or ſomewhere within the Primitive: And

I. An Angle of any Number of Degrees, when the Angular Point is at the Centre of the Primitive, will be deſcribed by taking the number of Degrees required in your Compaſſes, and ſetting it off in the Periphery of the Primitive, and from the Centre drawing two Radii to the Points found; for thoſe two Radii are the Repreſentations of two quadrantal Arks of two right Circles, and conſequently, the Meaſure of the Angle they make, will be the number of Degrees they intercept in the Limb, which is 90°. diſtant from the Angular Point.

II. If the Angular Point be at B a Point in the Periphery of the Primitive, and it were required to draw an Angle of 37°. thro' B (*See Fig. Pag.* 144.) and the Centre draw the Diameter BED, which croſs at right Angles with the Diameter AEC; the Diameter AEC produced, if occaſion be, will be the Line of Meaſures, with regard to the Circle, which will make the given Angle with the Primitive; and conſequently its Centre will be ſomewhere in this Line, and may be found thus; take the Secant of 37°. and ſet it from B to N, from which, and with the Radius BN, deſcribe the Circle BFD, and the Angle ABF will be the Angle requir'd, *viz.* an Angle of 37°. as is evident from the firſt *Prop.*

N. B. If the Angle given inſtead of 37°. had been an Angle of 143°. yet you muſt ſtill proceed after the ſame Manner, by taking the Secant of 37°. the Supplement of the Angle given, and therewith finding the Centre as before, and deſcribing the Circle, and the Angle FBC will be the Angle required, *viz.* an Angle of 143°.

III. If the Angular Point be any where within the Primitive but not at the Centre, here will be two Varieties of this third Caſe; for the Angle required to be made, may either be made by two oblique Circles, or by one oblique and one right One.

1, If it be required to make an Angle of 30°. by two oblique Circles, whoſe Angular Point ſhall be at P (ſee the Horizontal Projection) thro' P the Point given and the Centre Z, draw the Diameter NPZS, which croſs at right Angles with the Diameter WZE; then having the three Points W P and E, it will be eaſy to find K Centre of an oblique Circle, which ſhall paſs thro' thoſe three Points W P and E: Erect DB at right Angles to PK, and ſet off the Tangent of 30°. from K to S, and with the Radius SP deſcribe the Circle PH, and the Angle WPH will be the Angle required, made by two oblique Circles, and an Angle of 30°. as is evident from *n.* 5 of the Horizontal Projection.

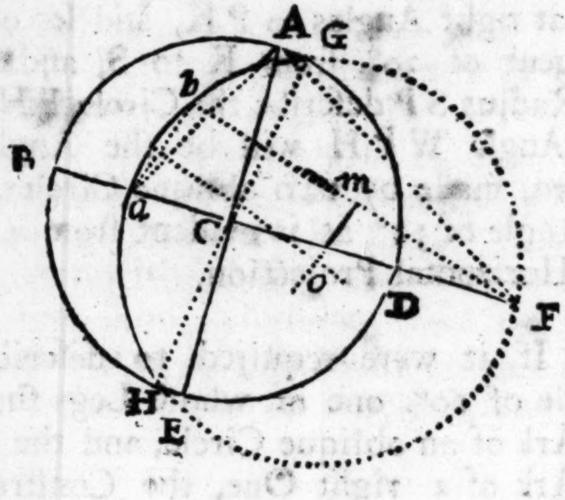
2. If it were required to deſcribe an Angle of 60°. one of whoſe Legs ſhall be the Ark of an oblique Circle, and the other an Ark of a right One, the Conſtruction

will be the ſame as before; only inſtead of the Tangent of the Angle required, its you muſt ſet the Tangent of Complement, *viz.* 30°. from K to S, and then with the Radius S P deſcribe the Circle P H; and the Angle H P S will be the Angle required, *viz.* an Angle made by the Ark of a right Circle and an oblique One, and an Angle of 60°. as you will perceive from *n.* 5. of the Horizontal Projection.

PROB. V.

To deſcribe the Repreſentations of a great Circle paſſing thro' two given Points.

Draw the Circle A B E D repreſenting



the Primitive, and let the Points be *a* and *b* take at Pleaſure; thro' any one of them as *a* and the Centre C, draw the Diameter BCD, which croſs at right Angles with the Diameter ACE; connect the given Point *a* with the Extremity of the other Diameter, and on the ſame ſide the Diameter BD that the Point *b* is, *i. e.* draw the prickt Line *a*A; and to the Point A at right Angle to *a* A, draw the prickt Line AF, which will interſect BD produced in the Point F: By this means you will gain a third Point as F, thro' which the Circle muſt paſs; but having three Points thro' which a Circle muſt paſs, it is eaſy from the Elements to deſcribe the Circle, as GFH*a*, which I thus prove to be the Repreſentation of a great Circle: To the two Points G and H where theſe two Circles, *viz.* the primitive and the oblique Repreſentation, interſect one another, draw the prickt Line GH; $GC \times HC$ is equal to $aC \times CF$ (*by* 38. *e.* 3.) which is equal to ACq (*by Cor.* 8. *e.* 6.) therefore $GC \times CH$ is equal to ACq, and conſequently GH paſſes thro' C the Centre of the Primitive; wherefore *a*GFH is the Repreſentation of a great Circle, Q. E. D.

BROB. VI.

To draw the Repreſentation of one Circle perpendicular to another, and firſt to draw one great Circle perpendicular to another great Circle.

Let it paſs thro' its Poles and 'tis done.

Here will be four Caſes,

I. The Repreſentation of a Circle perpendicular to the Primitive will be any one of its Diameters, for all of them repreſent right Circles paſſing thro' both its Poles, which are both Projected into the Centre.

II. The Repreſentation of a Circle perpendicular to a right One may be drawn by finding the Poles of that right Circle. and drawing a Diameter connecting the two Poles, which will be the Circle required: For inſtance, ACE (*See Fig. pag.* 154) being the Repreſentation of a right Circle, and B and D its Poles by *Prob.* 3. BCD will repreſent a right Circle perpendicular to ACE.

III. The Repreſentation of a Circle perpendicular to an oblique Circle may be drawn thus; firſt find the Poles of the right Circle by *Prob.* 3. then draw a great Circle thro' the two Poles found and 'tis done, by *Lemma* 2. for inſtance, the oblique Circle D E B is at right Angles to the right Circle ACE *(See Fig. pag.* 154.*)*

IV. To draw the Repreſentation of an oblique Circle perpendicular to another oblique Circle; ſuppoſe W Q E be the oblique Circle, to which it is required to draw an oblique Circle; that is perpendicular; firſt find the Pole P of the oblique Circle W Q E, and then through P and the two Points W and E draw the oblique Circle WPE, which will be the Repreſentation of a great Circle of the Sphere, and will be perpendicular to the former *Lemma.*

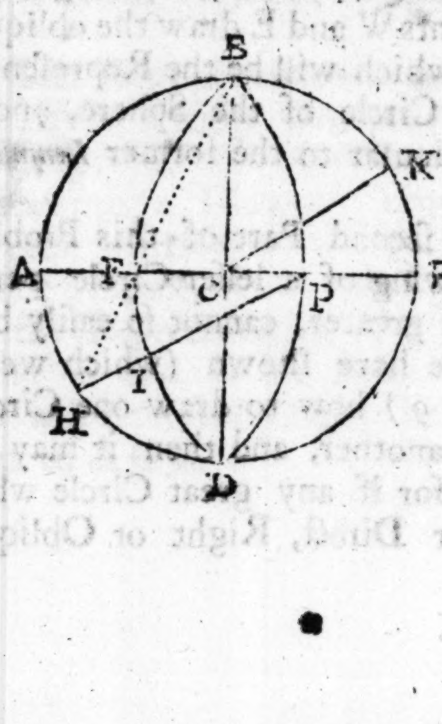
The ſecond Part of this Problem, *viz.* the drawing of a leſſer Circle perpendicular to a greater, cannot ſo eaſily be ſolved, until we have ſhown (which we ſhall do in *Prob.* 9.) how to draw one Circle parallel to another, and then it may be eaſily done; for if any great Circle whatſoever, whether Direct, Right or Oblique, were given, and it were required to draw a leſſer Circle perpendicular to it, 'tis but drawing a great Circle perpendicular to the great Circle given, which may be done by the former Part of this *Prob.* and then a leſſer Circle parallel to ſuch a greater Circle, and the Thing is done.

PROB. VII.

To meaſure the Quantity of any Ark of a great Circle in the Projection.

Here are three Caſes.

I. An Ark of a direct Circle, ſuppoſe BK, of the direct Circle ABED, will



be meaſured by fitting the Sector to the Circle A B E D, and then applying B K to the Line of Chords, which will give the Meaſure of the Ark B K.

II. The Portion of a right Circle, as F C (*See the foregoing Fig.*) of the right Circle A F C E, may be meaſured by taking F C in your Compaſſes, and applying it to the Scale of half-Tangents, for that will give the Number of Degrees which that Portion repreſents. The ſame Thing may be done another Way, thus; find B the Pole of the right Circle A F C E, and then through the two Points F and C draw the two Lines B F H and B C D, and H D apply'd to the Line of Chords, will give the Number of Degrees which F C repreſents; The Reaſon of both theſe Solutions is deducible from *Prop.* 1. This latter Method is more convenient in ſome Caſes.

III. The third Part of this *Prop. viz.*, the Meaſuring any Ark of an oblique Circle, is ſolved in the Corollary of the firſt *Prob.* which ſee.

PROB

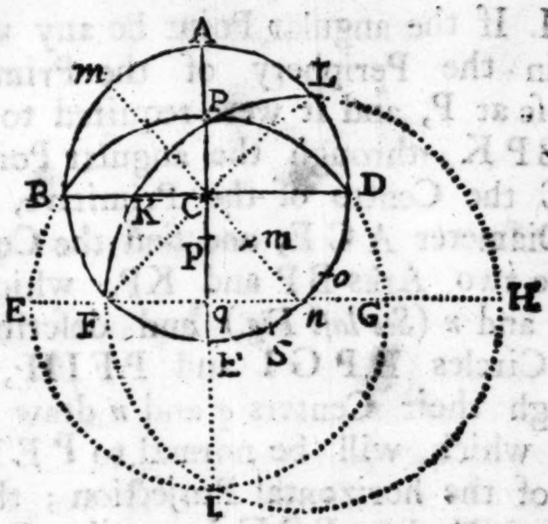
PROB. VIII.

To meaſure any Spherical Angle.

Here are three Caſes.

I. If the Angular Point be at the Centre of the Primitive, as the Angle B C K, (*See the foregoing Fig.*) this Angle may be meaſured by applying BK (the Ark intercepted between the Legs of the Angle) to the Line of Chords, which will give the Meaſure of the Angle.

II. If the angular Point be at the Periphery of the Primitive, (*See Fig. in the following Pag.*) as in the Angle A B P, thro' the angular Point B and the Centre C draw the Diameter B C D, which croſs at right Angles with the Diameter A C E, and then A P apply'd to the Scale of half-Tangents backwards from 90°, will give the Meaſure of the Angle, or apply'd to the Tangents backwards from 45°, doubling the Number of Degrees will do the ſame Thing. But if inſtead of A PB, it were required to meaſure the Angle PB C, C P apply'd to the half-Tangents according to their natural Order, will give the Meaſure of



of the Angle, or apply'd to the Tangents doubling the Number of Degrees. This is evident from *Prop.* 1. and *Def.* 2.

There is another Way of meaſuring a Spherical Angle from the Poles of thoſe two Arks which form the Angle, thus; in the Angle A B P, having found C and *p* the Poles of the two Circles B A D and B P D, through the angular Point B and the two Poles C and *p*, draw the two Lines B C D and B P O, and then O D apply'd to the Chords, will give the Quantity of the Angle A B P.

III. If the angular Point be any where within the Periphery of the Primitive; ſuppoſe at P, and it were required to meaſure B P K; through the angular Point P, and C the Centre of the Primitive, draw the Diameter A C E, and find the Centers of the two Arks B P and K P, which let be q and n (*See laſt Fig.*) and deſcribe the two Circles E P G I and P F I H, and through their Centers q and n draw E q G H, which will be normal to P E I by *n.* 5. of the horizontal Projection; therefore conſidering E P G I as the Primitive, and I F P H as an oblique Circle, the angular point P will be in the Periphery of the Primitive, and conſequently the Angle B P K as likewiſe F P q may be meaſured by the laſt Caſe.

This may be done another Way by finding the Poles of B P and K P, *viz.* p and m, in their proper Diameters A E and $m n$, and then laying a Rular through the angular Point P, and the the two Poles p and m, cutting the Primitive in the Points E and S; for E S apply'd to the Chords, will give the Meaſure of the Angle.

PROB.

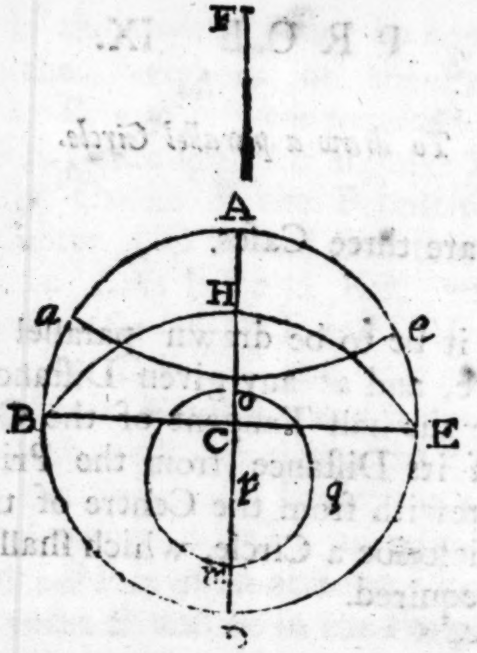
PROB IX.

To draw a parallel Circle.

Here are three Caſes.

I. If it be to be drawn parallel to the Primitive, and at any given Diſtance from it; take the half-Tangent of the Complement of its Diſtance from the Primitive, and therewith from the Centre of the Primitive deſcribe a Circle, which ſhall be the Circle required.

II. If it be to be drawn parallel to a right Circle, and at any given Diſtance from it; ſuppoſe it were required to draw a Circle parallel to B E at 30° Diſtance from it, ſet off the Chord of 30° from B to *n*, and from E to *e*, and then with the Tangent of the Complement of its Diſtance from the right Circle given, *viz.* 60°, placing the Compaſſes in *a* and *e*, ſtrike two croſſing Arks at F, and with the Diſtance *a* F deſcribe the Circle *a b e*,



a b e, which will be the Circle required, by *Prop*. 5.

III. If it be to be drawn parallel to an oblique Circle, and at any given Diſtance from it; let the great oblique Circle be B H E *(See the laſt Fig.)* and let the parallel Circle to be drawn be 50° diſtant from it; firſt find P the Pole of the oblique Circle by *Prob*. 3. and meaſure C *p* on the half-Tangents, and you will find it 28°, which ſubtract from 40°, the Complement of the given Diſtance, and there will remain 16°, the half-Tangent whereof ſet from C to *o*, will give one Extremity of its projected Diameter, and the ſemi-Tangent of 40° ſet from *c* to *m*, will give the other; and the Diſtance *o n* being biſſected, the Circle *o q u* may be deſcribed; which is the Circle required, as is evident from the foregoing Projection. Another Method of doing the ſame Thing may be eaſily deduced from *Prop.* 6 and 7.

PROB. X.

To Meaſure any projected Ark of parallel Circle.

I. If the Circle be parallel to the Primitive, then a Ruler laid through the Centre and the Diviſions in the Limb, will divide the Parallels into the ſame Degrees, or determin in the Limb the Quantity of any Ark parallel to it.

II. If the Circle be parallel to a right one, as *a b e* is to the right Circle B E, and it were required to meaſure any Part of it, or divide it into its proper Degrees, this will be done by *Cor. Prop.* 2. which you may conſult.

III. If the Circle be parallel to an oblique one, any Ark of it may likewiſe be meaſured by *Cor. Prob.* 2.

Spherick Trigonometry.

PART III.

DEFINITIONS.

I. A *Spherick Angle* is the mutual Aperture or Inclination of the Arks of two *great* Circles of the Sphere meeting in a Point, and is the ſame wit the Inclination of the Plains of thoſe Circles which conſtitute the Angle.

II. A *Spherick Triangle* is a Portion o the Surface of a Sphere, included betwee the Arks of three great Circles of th Sphere.

III. The Quantity of a Spherical Angle is eſtimated or meaſured by an Ar of a great Circle intercepted between th two Arks which conſtitute the Angle at 90 diſtant from the angular Point.

Proper

Properties of Spherical Triangles.

I. When one great Circle falls upon another great Circle, the Sum of the contiguous Angles is equal to two right ones; becauſe their Meaſure is a Semi-circle.

II. When one great Circle croſſes another, the vertical Angles made thereby are mutually equal, as being meaſured by equal Arks.

III. The greater Angle is always ſubtended by, or oppoſed to, the greater Ark or Side.

IV. An Iſoſceles Triangle has its two Angles at the Baſe equal to one another; and on the contrary, if a Triangle has two Angles equal, the ſubtending Sides or Arks will be alſo equal.

V. Two Triangles mutually Equilateral are alſo mutually Equiangular to one another

VI. If in two Triangles one Angle and the two including Sides be reſpectively equal; or if one Side and the Angles adjacent be ſeverally equal, then the two

Tri-

Triangles are equal; for if laid upon one another they will perfectly concur or cover one another.

VII. Two Sides of a Spherical Triangle are always greater than the Third; for the Ark of a great Circle is the neareſt Diſtance between two Points on the Surface of a Sphere, as a ſtreight Line is betwixt two Points in a Plain.

Theſe ſeven Properties are common plain Triangles, and have a like Demonſtration,

PROB. VI.

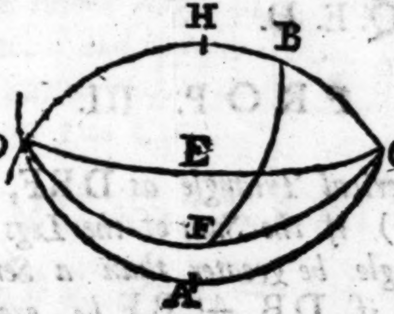
All great Circles of the Sphere cut one anothe into two equal Parts.

DEMONSTRATION.

The Plains of all great Circles upon th Sphere paſs through the Centre, and con ſequently, the common Interſection of an two great Circles will be a Diameter com mon to both the interſecting Circles; bu a Diameter (from Geometry) biſſects th Circle or Circles to which it belongs: There fore all great Circles, *&c.* Q. E. D.

COROL.

From hence it follows, that every Side



of a Spherical Triangle is leſs than a Semi-circle; for if one Side be a Semi-circle as DBC (*in the foregoing Fig.*) then the Ark of any great Circle as DE, DF, or DA, making any Angle with DC, will, being produced, interſect DC at a Diameter's Diſtance in C, and conſequently can never form a Triangle.

PROP. II.

The oppoſite Angles at the Sections of two great Circles are equal, i.e. *The Angle* HDA = *Angle* HCA.

DEMONSTRATION.

By *Def.* 3. the Ark of a great Circle intercepted between H and A, (*See the laſt Fig.*)

Fig.) the two quadrantal Points of the Semi-circles D H G and D A C, will be the Meaſure of the Angles at D and C, and conſequently (the ſame Ark being the Meaſure of two Angles) the Angle H D A = Angle H C A. Q. E. D.

PROP. III.

In any Spherical Triangle as D B F, (See *Fig. Prop.* I.) *if the Sum of the Legs including any Angle be greater than a Semi-circle ; that is, if* D B + B F *be greater than* D B C, *than the intern Angle at the Baſe* D *is greater than the outward oppoſite Angle* B F C, *and the two Internal Angles at the Baſe* B D F *and* D F B *are greater than two* ∟s.

DEMONSTRATION.

D B + B F being greater than the Semi-circle D B C, the Side B F is greater than B C, and conſequently (by *Property* 3.) the Angle B C F = B D F is greater than BFC; and B D F + D F B is greater than the Angles D F B + B F C ; but D F B + DFC are equal to two ∟s ; therefore BDF + D F B are greater than two right Angles. Q. E. D.

PROP.

PROP. IV.

If DB + BF *be* = *the Semi-circle* DBC, *then the intern Angle at the Baſe* D *is* = *the outward and oppoſite Angle* BFC; *and the two intern Angles at the Baſe* BDF + DFB = *two* ∟*s.*

DEMONSTRATION.

DB + BF being = the Semi-circle DBC, the Side BF is = BC, and conſequently (by *Property* 3.) the Angle BCF = BDF (by *Prop.* 2.) is = BFC; and BDF + DFB = DFB + BFC; but DEB + BFC is = two ∟s; wherefore BDF + DFB is = two ∟s. Q.E.D.

PROP. V.

If DB + BF *be leſs then the Semi-circle* DBC, *than the intern Angle at the Baſe* D *is leſs than the outward and oppoſite Angle* BFC; *and the two intern Angles at the Baſe* BDF + DFB *are leſs than two right ones.*

The Demonſtration proceeds juſt after the ſame Manner with the two former, and need not be here inſerted.

COROLL.

Hence it is plain, that in an Iſoſceles Triangle if one of the equal Legs be greater, equal, or leſs than a Quadrant, the Angle at the Baſe is accordingly greater, equal, or leſs than a right Angle.

PROP. VI.

The Sum of the three Sides of any Spherical Triangle are leſs then a Circle; that is, D B + B F + F D *is leſs than* D B C + D F C.

DEMONSTRATION.

In the Triangle B F C the Side B F is leſs than B C + F C (by *Property* 7.) and adding D B + D F to both Sides D B + D F + B F, the Sum of the Sides of the △ D B F is leſs than D B C + D F C. Q. E. D.

PROP

PROP. VII.

The Poles of the Sides of any Triangle GHD *conſtitute another Triangle* n x m, *which we may call ſupplemental to the Triangle* GHD; *for the Supplements of the Angles of the Triangle* n x m *are equal to the Sides of the Triangle* GHD, *and the Supplements of the Sides of the Triangle* n x m *are equal to the Angles of the Triangle* GHD.

DEMONSTRATION.

From the angular Points of the Triangle GHD as Poles, deſcribe on the Globe 3 great Circles; that is, finding the Centre of any of the Arks GD, HD or GH, and ſetting the Sector to the Radius, deſcribe with the Chord of 60°, the great Circles x A y, R T $m n$, x B n Z; then m being the Pole of GH, m Y = Quadrant (the Diſtance between the Pole of any great Circle, and the Periphery of that great Circle being always = Quadrant) = A x, x or E being the Pole of the Ark GD; therefore adding m A on both Sides $m x$ = A y = Supplement of CA, which CA is the Meaſure of the Angle HGD.

n Being the Pole of HD, zn = Quadrant = Bx, and adding Bn on both Sides nx = Bz = Supplement of B K, which is the Meaſure of the Angle H D G.

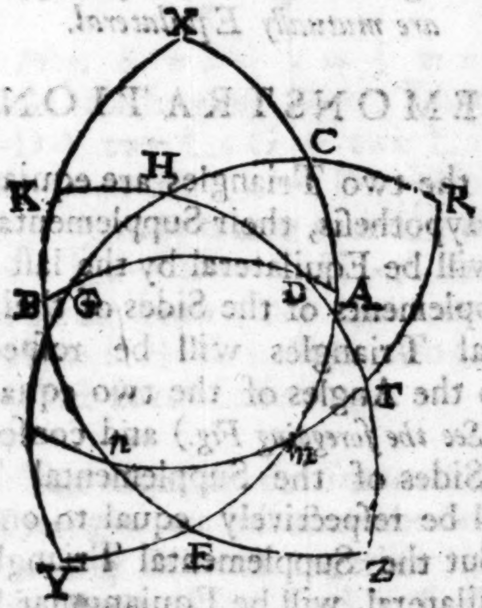
mn = T R, *i. e.* the Diſtance between the Poles of any two great Circles, is equal to the Inclination of thoſe Circles. But n being the Pole of H D, and m the Pole of G H, mn will be the Diſtance between the two Poles, and the Angle D H R is the Inclination of the two Circles, whoſe Meaſure is T R; wherefore mn = T R, and adding m T on both Sides mR = T n; but m being the Poſe of G H, m R = Quadrant; therefore T n is a Quadrant alſo. nm being = T R, nm will be equal to the Meaſure of the Supplement of the Angle G H D. So that the latter Part of the Propoſition is proved. That is the Supplements of the Sides of the Triangle nxm = Angles of the Triangle G H D.

The Truth of the firſt Part may be proved thus.

The Triangle n E m conſtituted between the three next Poles hath its three Sides equal to the Angles, and its three Angles equal to the Sides of the Triangle G H D; ſave that the greateſt Side nm is the Supplement of the Angle H, and the Angle E of the Side G D.

PROP. VIII.

Any Angle of a Triangle with the Difference of the other two, is leſs than two right Angles, i. e. $G + H - D$ ∟ 2 ∟s.



DEMONSTRATION.

In the Triangle xum, $xn < xm + mn$ (by *Prop.* 7.) that is two ∟s − D (nE being equal to BK) < two ∟s − G = Em + two ∟s − H (for by the laſt mn = Supplement of H) and taking two Right An-

Angles from each Side — D ∟. two ∟s — G — H, and adding G and H on both Sides G + H — D < two ∟s Q. E. D.

PROP. IX.

If two Triangles are mutually Equiangular, they are mutually Equilateral.

DEMONSTRATION.

Since the two Triangles are equiangular by the Hypotheſis, their Supplemental Triangles will be Equilateral by the laſt; ſince the Supplements of the Sides of their Supplemental Triangles will be reſpectively equal to the Angles of the two equal Triangles (*See the foregoing Fig.*) and conſequently the Sides of the Supplemental Triangles will be reſpectively equal to one another: But the Supplemental Triangles being Equilateral, will be Equiangular by the Fifth *Property*, and being Equiangular, the Sides of the two Equiangular Triangles given, will be alſo reſpectively equal, as being equal to the Supplements of the two Equilateral Supplemental Triangles by *Prop.* 7. Q. E. D.

PROP. X.

The three Angles of every Spherick Triangle, are greater than two Right ones, and leſs than ſix right ones.

DEMONSTRATION.

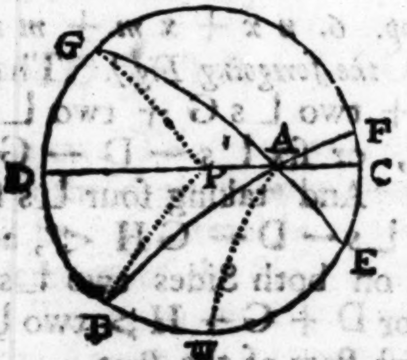
By *Prop.* 6. $n x + x m + m n <$ four ∟s, (*See the foregoing Fig.*). That is, two ∟s $- D +$ two ∟s $G +$ two ∟s $- H <$ four ∟s, or ſix ∟s $- D - G - H <$ four ∟s. And taking four ∟s from each Side two ∟s $- D - G H <^{\circ}$, and adding D, G, H on both Sides two ∟s $< D + G + H$, or $D + G + H >$ two ∟s, which is the Firſt Part of the *Prop.*

II. The Sum of the Internal Angles of any Triangle, are leſs than the Sum of the Internal and External together; all which are (by *Proper.* 1.) = ſix ∟s. E. C. P.

PROP.

PROP. IX.

Of ſeveral Arks of great Circles falling from the ſame Point of the Sphere's Surface on another Circle, the greateſt is that which paſſes thro' the Pole of the Circle.



DEMONSTRATION.

Let P be the Pole of the Circle $C\varpi E$, and ϖ the Pole of the Circle DPC, then will AD ($= AP + DP$, or BP) be $> AB > AE < AC$, and the Ark $B\varpi G > BP > BD$.

PROP. XII.

A great Circle paſſing thro' the Poles of another great Circle, cuts it at Right Angles; and on the Contrary, if it cuts it at Right Angles, it paſſes thro' its Poles.

This is Demonſtrated in *Lemma* 2 of the *Spherick Geometry*; which See.

PROP. XIII.

In an oblique-angled Triangle, if the Angles at the Baſe are like, that is, both Acute, or both Obtuſe; The Perpendicular let fall from the Oppoſite Angle, falls within the Triangle, and the Quadrantal Ark without: But if they be unlike, the Perpendicular fall without and the Quadrant within.

For in the Triangle EAF, whoſe Angles at E and F are both Acute, the Perpendicular AC falls within, and the Quadrant A π without: Alſo in the Triangle BAG, whoſe Angles at B and G are Obtuſe, the Perpendicular AD falls within, and the Quadrant A π without.

But in the Triangle BAE, whoſe Angles at B and E are of different kinds, the

Perpendicular A C falls without, and the Quadrant A * within.

By the ſame Figure may the Ambiguity of Right Angled Triangles be ſolved.

Solutions of the Ambiguity of Right Angled Triangles.

I. The Legs of the Right Angle are of the ſame kind with the Oppoſite Angles; ſo in the Triangle BDA, (*See the foregoing Figure, Pag.* 174.) Right Angled at D by *Prop.* 12. becauſe D A is ⌟ Quadrant DP, the Angle DBA is ⌟ than the Right Angle DBP (by *Proper.* 3.) And in the Triangle BCA, becauſe A C is ⌞ Quadrant P C, the Angle CBA is leſs than the Right Angle CBP.

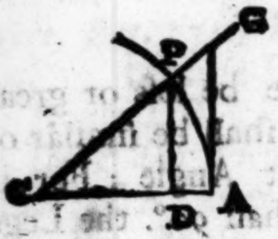
II. If the Legs, and conſequently the Angles (*by the Firſt*) are of the ſame, or of different kind, the Hypothenuſe is accordingly leſs or greater than a Quadrant. So in the Triangles D A E and E A C, the Legs D A and D E being greater, and A C and EC being leſs than a Quadrant, (which is what is meant by being of the ſame kind) the Hypothenuſe A E is leſs than a Quadrant. But in the Triangle BDA, whoſe Legs DA and DB are of different kind, to wit DA greater, and DB leſs than a Quadrant; the Hypothenuſe AB is ⌟ than the Quadrant DP.

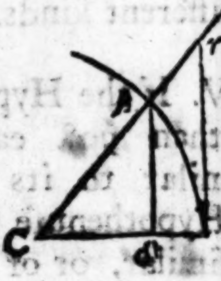
III. On the contrary, if the Hypothenuſe be leſs than a Quadrant, or 90°. then are the Legs, as alſo the two oblique Angles are ſimilar, or of the ſame kind; but if greater than a Quadrant, then are the Legs and the two oblique Angles diſſimilar, or of different kinds.

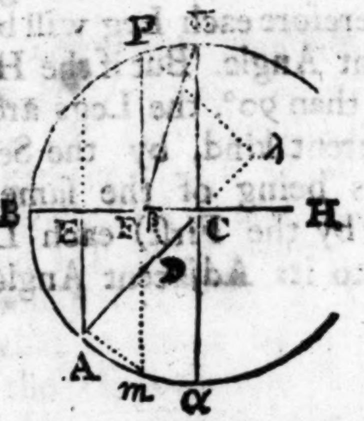
IV. If the Hypothenuſe be leſs or greater than 90°. each Leg ſhall be ſimilar or diſſimilar to its Adjacent Angle: For if the Hypothenuſe be leſs than 90°. the Legs are ſimilar, or of the ſame kind, by Solution Second; as alſo the Angles by the Firſt; wherefore each Leg will be ſimilar to its Adjacent Angle. But if the Hypothenuſe be greater than 90°. the Legs are diſſimilar, or of different kind, by the Second; and the Angles being of the ſame kind with the Legs (by the Firſt) each Leg will be diſſimilar to its Adjacent Angle, and *vice verſâ.*

For viewing the Sines, Coſines and other Right Lines of Arks which are not viſible in a Common Sphere, 'tis convenient to uſe the Planes of the Arks of three great Circles of Card and Paſte-board put together, as in an Armillary Sphere.

As ſuppoſe the two Arks BP, BA, and that BPH the Plane of the greater Ark





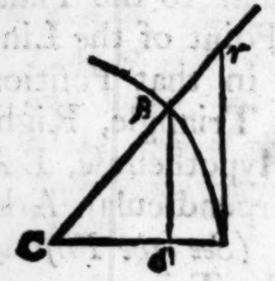


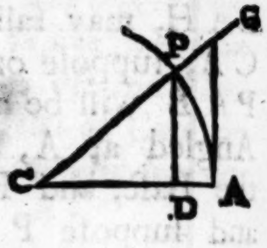
were turn'd round BH, till a Right Line

falling from P perpendicular to the Plan BAH, may fall on ſome Point of the Line CA, ſuppoſe on D; for in that Poſition PAB will be a Spherick Triangle, Right Angled at A, BP the Hypothenuſe, BA the Baſe, and PA the Perpendicular Ark, and ſuppoſe PA = PA (*See the Firſt of the three foregoing Fig.*) of the Triangle, and fitted according to its Letters therein; draw AE, PF perpendicular to BC; then then will AE, PF and PD be the Sines of the Arks BA, BP and PA, and their Coſines will be EC, FC and DC. Theſe things being done or conceiv'd, the two Firſt Axioms of *Spherick Trigonometry* will preſently appear; and conſequently, the demonſtration of the Sixteen Caſes of Right Angled Triangles. To which end let $\pi\alpha$ be fitted in the Solid, according to its Letters than in the two Right-angled Spherick Triangles PBA, $\pi B\alpha$, having the ſame Acute Angle at the Baſe.

AXIOM I.

In all Right-angled Triangles, having the ſame or equal Acute Angles at the Baſe, the Sines of the Hypothenuſes are proportional to the Sines of the Perpendiculars, viz. $PF : PD :: \pi C : \pi\delta$: *From Sim. Triangles.*

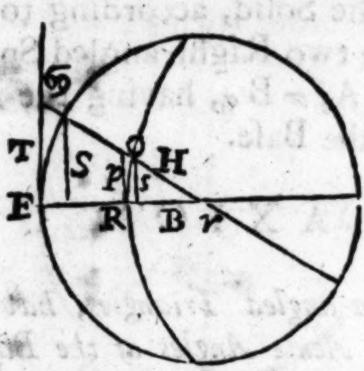




AXIOM II.

In all Right angled Triangles, having the ſame, or equal Acute Angles at the Baſe, the Sines of the Baſes are proportional to the Tangents of the Perpendicular, viz. AE : AG :: aC : aγ. *(See Fig. Pag.* 177.*)*

The Firſt and Second *Axioms* may be Demonſtrated alſo by this Diagram.



Axiom I. S, H : S, ♋ ♈ :: s : S : } from Sim:
Axiom II. S, B : S, E ♈ :: P T : } Triangles.

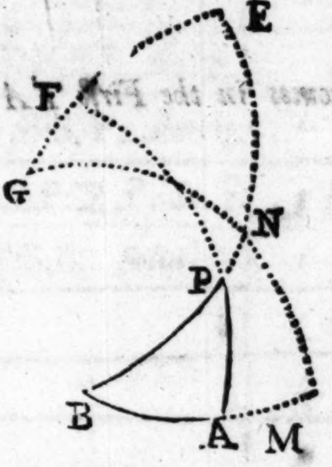
Theſ

Theſe two *Axioms* may be as well interpreted without the Ark π α, and their congruity with plain Trigonometry appear, by only conſidering the Plain Right angled Triangles F D P and E A G; for,

I. PF : R : : PD : S, P F D : *i. e.* S, of Hypothenuſe. R : : S of Perpendicular. S, of the Angle at the Baſe.

II. A E : R : : A G : T, A E G = B, *i. e.* S of Baſe. R : : T, of the Perpendicular. T, Angle at the Baſe.

For the following Caſes, I ſuppoſe B A P a Right angled Triangle, and its Sides produc'd to Quadrants B M, B N and A D. Suppoſe alſo P E, P F, N G and E G Quadrants;



drants; then is NE = BP, and the Complement of BA = AM = Angle ADM, AD and MD being both Quadrants. Alſo EF = Angle FPE = Angle BPA, and GD = NM = Angle at B, and the Angles at A, M, N, E, F, right (*By Prop.* 12.)

N. B. The Reaſon of producing theſe Sides of the Triangle to Quadrants, is becauſe by this means the Angles are turn'd into Sides, and the Hypothenuſes into Baſes and Perpendiculars, *& e contra*; whence the Proportions as to the Parts of the Triangle given, do ſometimes fall in Co-ſines and Co-Tangents, inſtead of Sines and Tangents.

(*Here comes in the Firſt TABLE.*

Given	Req.	*The Proportions of the 16 Caſes of right-angled Triangles, with the Solutions of their Ambiguities.*	Given	Req.	Nº
BA, PA	BP.	S, DAS:AM::S,DP:S,PN : *by Axiom* 1. *i. e.* R:Σ,BA::Σ,PA:Σ,BP. *Sol.* 2.	*c c*	*b*	1.
BA, PA	B.	S,BA:S,BM:T,P A:T,NM: *by Axiom* 2. *i. e.* S, BA:R::T,PA:T,B. *Solut.* 1.	*c c*	∟	2.
BP, P	B.	S,PE:S,PN::T,EF:T,ND : *by Axiom* 2. *i. e.* R : Σ,BP::T,P: τ, B. *Solut.* 1.	*b* ∟	∟	3.
BP, P	P A.	S,GE:S,GF::T,EN:T,FD: *by Axiom* 2. *i. e.* R : Σ,P::T,BP:T,PA: *Solut.* 3.	*b* ∟	∟ *adj.*	4.
BP, P	B A.	S,BN:S,BP::S,FE:S,BA. *by Axiom* 1. *i. e.* R:S,BP::S,P:S,BA. *Solut.* 1.	*b* ∟	*c, opp.*	5.
PA, P	B P.	S,GF:S,GE::T,FD:T,EN, *by Axiom* 2. *i. e.* Σ, P:R::T,PA:T,BP. *Solut* 3.	*c,* ∟ *adj.*	*b*	6.
PA, P	B.	S,PF:S,PD::S,EF:S,DN, *by Axiom* 1. *i. e.* R : Σ, PA::S,P:Σ,B. *Solut.* 1.	*c,* ∟ *adj.*	∟	7.
BA, B	P A.	S,BM:S,BA::T,MN:T,AP, *by Axiom* 2. *i. e.* R:S,BA::T,B:T,AP. *Solut.* 1.	*c,* ∟ *adj.*	*c*	8.
PA, B	B A.	T,MN:S,MD=BM::TPA:S,BA, *by Ax.* 2. *i. e.* T,MN:R:T,PA:S,BA. *Am.*	*c,* ∟ *opp.*	*c*	9.
PA, B	B P.	S,NM:S,AP::S,MD=BN:S,BP, *by Axiom* 1. *i. e.* S,B:S,AP::R:S,BP. *Ambig.*	*c,* ∟ *opp.*	*b*	10.
PA, B	P.	S,PD:S,PF::S,DN:S,FE, *by Axiom* 1. *i. e.* Σ,AP:R::Σ,B:S, P, *Ambig.*	*c,* ∟ *opp.*	∟	11.
PA, BP	P.	T,NE:S,GE=PE::T,DF:S,GF, *by Ax.* 2. *i. e.* T,BP:R::T,AP:Σ,P, *So.* 3.	*c b*	∟*adj.*	12.
PA, BP	B.	S,BP:S,BN::S,AP:S,NM, *by Axiom* 1. *i. e.* S, BP:R::SAP:S,B. *Solut.* 1.	*c b*	∟*opp.*	13.
PA, BP	B A.	Σ, PA:R::Σ, BP:Σ, BA (*by Caſe* 1.) *Solution* 2.	*c b*	*c*	14.
B, P	B P.	τ, P:R: τ,E:Σ,BP. (*by Caſe* 3.) *Solution* 2.	∟ ∟	*b*	15.
B, P	P A.	S,P : R::Σ, B:Σ, AP. (*by Caſe* 11.) *Solution* 1.	∟ ∟	*c*	16.

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N. B. By theſe Sixteen Proportions, together with the Three foregoing Solutions, may Thirteen of the Sixteen Caſes of Right angled Triangles, together with their Ambiguities be eaſily ſolv'd: And as for the remaining Three, which are the Ninth, Tenth, and Eleventh in the preceeding Table; tho' you have Proportions there for their Solution, yet they ſtill remain Ambiguous, as falling under none of the foregoing Solutions; ſo that you muſt determine from other Circumſtances of the Problem propos'd, whether an Ark or its Supplement be requir'd, ſince both of them have the ſame Sine.

Aſtronomical Problems;

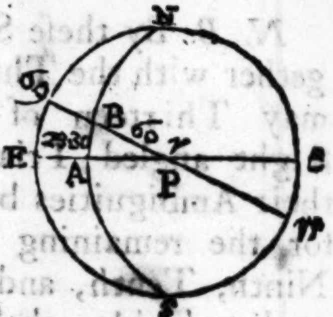
Being the uſeful CASES of Right angled Spherick *Triangles.*

CASE I.

Given PB *the* ☉'s *Longitude, and* P *his greateſt Declination; Required his Right Aſcenſion or* P A.

By the Fourth Caſe. (*See Table. Pag.* 148)

$$\boxed{R : \Sigma P :: T,BP : T,PA}$$



BP = 60°.
P = 23°. 30′.

R = 10.

Σ, P = 9.962398
T,BP = 10.238561

T,PA = 10.200959 = 57°. 48′

CASE II.

Given BP *the* ☉'*s Longitude, and* P *his greateſt Declination*; *Required his preſent Declination*, viz. BA.

By the Fifth Caſe, (*See Fig. Pag.* 184.)

R : S,BP :: S,P : S,BA

R = 10

S,BP = 9.937531
S,P = 9.600700

S,BA = 9.538231 = 20°. 12′

CASE III.

Given P A *the Poles Height, and* B A *the Complement of the* ☉'*s Amplitude; Required the time from Midnight, or* P.

By the Second Caſe, (*See Fig. Pag.* 185.)

$$\boxed{S,PA : R :: T,\overline{BA} : T,P}$$

P A = 51°. 30′
B A = 56°. 40′

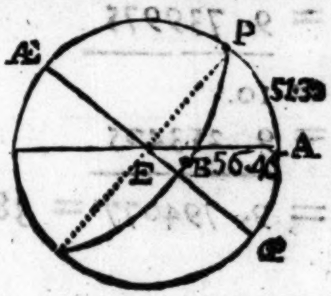
S, P A = 9. 893544

R = 10.
T,B A = 10. 181965

T, P = 10. 288421 = 62°. 46′

CASE IV.

Given the ſame Data; Required P B *the Complement of the Sun's Declination.*



By Caſe the Seventh,

$$\boxed{R : \Sigma, BA :: \Sigma PA : \Sigma, BP}$$

$R = 10.$

$\Sigma, BA = 9.739975$
$\Sigma, PA = \underline{9.794150}$

$\Sigma, BP = 9.534125 = 20^\circ.$

CASE V.

Given P B *and* B A; *Required the Latitude or Pole's Height* = P A.

By Caſe the 14th, (*See Fig. Pag.* 185.)

$$\boxed{\Sigma, BA : R :: \Sigma, BP : \Sigma PA}$$

$\Sigma, BA = \underline{9.739975}$

$R = 10.$
$\Sigma, PB = \underline{9.253405}$

$\Sigma, PA = 9.794077 = 38^\circ. 30'$

CASE

CASE VI.

Given P A *the Pole's Height, and* P B *the Complement of the Sun's Declination; Required the Complement of his Amplitude* = B A.

By Caſe the 14th, (*See Fig. Pag.* 185.)

$$\boxed{\Sigma, PA : R :: \Sigma, BP : \Sigma, BA}$$

$PA = 51^\circ.\ 30'$
$BP = 70^\circ.$

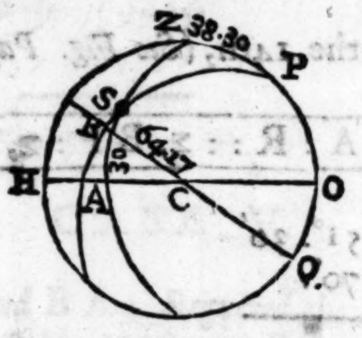
$\Sigma, PA = 9.\ 794151$
$R = 10.$
$\Sigma, BP = 9.\ 534052$

$\Sigma, BA = 9.\ 739901 = 33^\circ.\ 20'$

CASE

CASE VII.

In Latitude 51° 30′, *the Sun being in the Æquator, and it being two Hours paſt Noon; Required the Sun's Altitude.* = A ☉. *'Tis to be obſerved that the Point* S *or* ☉ *ſhould be in* ÆQ.



By Caſe the Firſt.

$$\boxed{R : \Sigma, \AE\odot :: \Sigma, ZE : \Sigma, Z\odot}$$

$\AE Z = 51^\circ. 30'$
$\AE\odot = 30^\circ.$

$R = 10.$

$\Sigma, \AE\odot = 9.937531$
$\Sigma, Z\AE = 9.794150$

$\Sigma, Z\odot = 9.731681 = 32^\circ. 37' =$ A ☉.

N. B. The ſame Diagram will ſerve alſo for finding the Azimuth, when the Sun is in the Equator, by having his Altitude.

For, Given Æ Z *the Co-Altitude, and* B Z *the Complement of the Sun's Altitude. Required the included Angle at* Z.

By Caſe the Twelfth,

$$\boxed{T,BZ : R :: T,EZ : \Sigma,Z}$$

$T,BZ = 10.193863$

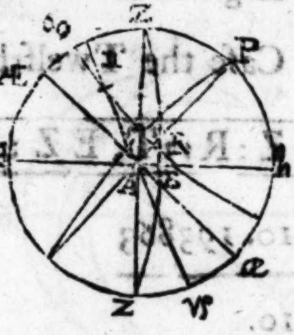
$R = 10.$

$T,ÆZ = 10.099395$

$\Sigma, Z = 9.905532 = 53°.34'$

CASE VIII.

In the Right-angled Triangle A R B, *given the Angle* A = *the Pole's Height, and* A R, *the Sun's Declination; Required* R B *the Sun's Altitude at the Hour of Six.*



By Caſe the Fifth,

$$\boxed{R : S, \underline{RA} :: \underline{S, A} : \underline{S, RB}}$$

Pole's Height = A = 51°. 30′
A R = 20°. 12′

R = 10

S, A R = 9. 539223
S, A = 9. 893544

S, R B = 9. 432767 = 15°. 43′

CASE

CASE IX.

In the Triangle Z ⊙ P, *the Sun being due Eaſt or Weſt, there is given* ⊙ P, *the Co-declination, and* Z P, *the Co-latitude ; Required the Angle* Z P ⊙, *or the Hour from Noon.*

By the 12th Caſe, *(See Fig. Pag.* 190*)*

$$\boxed{\overline{\underline{T,P\odot}} : \overline{\underline{R}} :: \overline{\underline{T,PZ}} : \overline{\underline{S,P}}}$$

$$\odot P = 69^{\circ}.48'$$
$$ZP = 38^{\circ}.30'$$

$$T,P\odot = 10.433068$$

$$R = 10.$$
$$T,PZ = 9.900605$$

$$S,P = 9.467537 = 17^{\circ}.4.$$

CASE X.

Given the ſame Data; Required his Altitud when he's due Eaſt or Weſt.

By the 14th Caſe, (*See Fig. pag.* 190.)

$\boxed{\Sigma, PZ : R :: \Sigma, \odot P : \Sigma Z \odot}$

$\Sigma\ PZ = 9.893544$

$R = 10.$

$\Sigma, \odot P = 9.539223$

$\Sigma, \odot Z = 9.645679 = 26°. 15'.$

CASE

CASE XI.

In the Triangle Z R P, *Given* Z P *the Co-Latitude, and* R P *the Co-Declination, Required the Angle* R Z P *or* B N *the* ☉*'s Azimuth from the North at the hour of Six.*

By the Second Caſe, (*See Fig. Pag.* 184.)

$$\boxed{S,ZP : R :: T,RP : T,Z.}$$

$$S,ZP = 9.294150$$

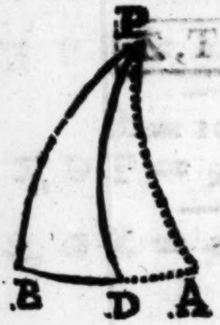
$$R = 10.$$

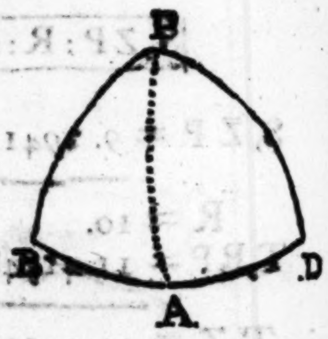
$$T,RP = 11.415486$$

$$T,Z = 12.121336 = 89^{\circ}.\,34'.$$

The Solution of oblique Spherick Triangles.

Oblique Spherical Triangles may be reduced to two Right-angled Spherical Triangles, by letting fall a Perpendicular, which Perpendicular either divides the oblique Triangle propoſed, into two right-





angled Triangles, or makes two right-angled Triangles, by adding a right-angled Triangle to it, and then theſe Rules will ariſe for their Solutions.

In oblique angled Triangles, there are twelve Caſes, Ten of which (being thus prepared) may be ſolv'd by the Two firſt *Axioms*, or Rules deduced from them.

RULE I.

The Coſines of the Angles at the Baſe are proportional to the Sines of the Angles at the Vertex, (*See foregoing Fig.*) For (by Caſe 7. of right-angled Triangles.)

Σ,B:S,BPA::Σ,PA:R ⎫ And *ex æquo*
And ⎬ Σ,B:Σ,D::S,BPA:
Σ,D:S,DPA::Σ,PA:R ⎭ (S,DPA:

RULE II

The Co-ſines of the Sides are Proportional to the Co-ſines of the Baſes: For (by Caſe 1. of Right-angled Triangles.)

Σ,BA:Σ BP:: R:Σ,PA ⎫ And *ex æquo*
And ⎬ Σ,BA:Σ,DA::Σ
Σ,DA:Σ DP::R:Σ,PA ⎭ (BP:Σ,DP:

RULE III.

The Sines of the Baſes are reciprocally proportional to the Tangents of the Angles at the Baſe. For (by *Axiom* 2),

S,BA:R::T,PA:T,B ⎫ And *ex æquo*
And ⎬ S,BA:S,DA::T,D:T,
S,DA:R::T,PA:T,D ⎭ (B

RULE IV.

The Tangents of the Sides are reciprocally proportional to the Co-ſines of the Angles of the Vertex. For (by Caſe 4 of Right-angled △'s.)

T,BP:T,PA ::R,Σ,BPA ⎫ *Ergo*
And ⎬ T,BP:T,DP::Σ,
T,DP:T,PA::R:Σ,DPA ⎭ (DPA:Σ,BPA.

AXIOM III.

In any Triangle, the Sines of the Sides are proportional to the Sines of the oppoſite Angles.

DEMONSTRATION.

By *Axiom* 1. S,BP:R::S,PA : S,B. And S,DP:R::S,PA:S,D *ergo* S,BP:S,DP::S,DS,B. Q. E. D.

(Here comes in the Second TABLE.

Given	*Req*	*The Proportions of the 12 Caſes of oblique angled Triangles.*	*Given.*	*Req*	Nº
BP,PD,B	D	S, PD : S, B : : S, BP : S, D, *Ambiguous.*	2 *l.* ∠ *opp.*	∠ *op.*	1.
PB, B, D	DP	S,D : S, B P : : S, B, S, P D, *Ambiguous.*	2 ∠ *l. opp.*	*l. op.*	2.

The 8 following Caſes are reſolv'd by letting fall from the Extream of a given Side a Perpendicular oppoſite to a given Angle (See Fig *p.* 194.) *And you muſt obſerve the Addition or Subtraction both of the Segments of the Baſe, and the Angles at the top, according as the Perpendicular falls within or without the Triangle.*

BP,PD,B	BD	R:Σ, B :: T,BP,T,BA; *(*By Caſe 4. of ∟d △'s*)* and then by Rule 2. Σ,BP, Σ,BA::Σ,DP:Σ,DA. But here it is doubtful whether the Perpendicular falls within or without the △ unleſs the kind of the ∠ D is foreknown.	2 *l,* ∠ *opp.*	*l*	3.
BP,PD,B	P	R:Σ,BP::T,B:τ,BPA, (By Caſe 3.) and then by Rule 4. T, DP:T, BP::Σ, BPA:Σ,DPA. Here alſo the falling of PA is doubtful, unleſs you know the kind of the Angle D.	2 *l,* ∠ *op.*	∠ ↄ *p.*	4.
BP, B, D	P	R:Σ,BP::T,B:τ,BPA, (By Caſe 3.) and by Rule 1. Σ, B:Σ, BPA::Σ,D: S, DPA. If D, B are like the Sum of BPA, DPA = P, elſe their Difference = P.	2 ∠, *l. op.*	∠	5.
BP, B, D	BD	R:Σ,B:: T, BP:T,BA (By Caſe 4.) and by Rule 3.T,D:T,B::S,BA:S,DA. If B is like D, the Sum of BA, DA=BD, elſe their Diff. is = BD.	2 ∠, *l. op.*	*l* ↄ *pr.*	6.
B, P, BP	D	R:Σ,BP::T,B,τ,BPA*(*By Caſe 3*)* and by Rule 1.S.BPA:S,DPA::Σ,B:Σ D. If BPA is greater then BPD and alſo B$\left\{\begin{array}{l}\text{Acute,}\\\text{Obtuſe,}\end{array}\right\}$D is$\left\{\begin{array}{l}\text{Obtuſe,}\\\text{Acute,}\end{array}\right.$ But if BPA > BPD and B$\left\{\begin{array}{l}\text{Acute,}\\\text{Obtuſe,}\end{array}\right\}$D is$\left\{\begin{array}{l}\text{Acute,}\\\text{Obtuſe.}\end{array}\right.$	2 ∠, *l.* ↄ *pr.*	*l*	7.
B, P, BP	DP	R:Σ,BP::T,B:τ,BPA. (By Caſe 3.) & by Rule 4. Σ,DPA:Σ,BPA::P,BP: T,DP. If DPA is like or unlike B;DP is leſs or greater than a Quadrant.	2 ∠, *l.* ↄ *pr.*	*l. opp.*	8.
BP,BD,B	DP	R: Σ,B::T,BP:T, BA. *(*By Caſe 4.*)* and by Rule 2. Σ, BA:Σ,BP::M, DA:Σ,DP. If DA is like or unlike (PA) ∠ B, then PD is > < 90°.	2 *l.* ∠ ↄ *pr.*	*l.*	9.
BP,BD,B	D	R:Σ,B::T,BP:T,BA. *(*by Caſe 4.*)* and by Rule 3. S, DA:S, BA :: T, B: T,D. If BA< > BD, D will be like unlike B.	2 *l.* ∠ ↄ *pr.*	∠	10

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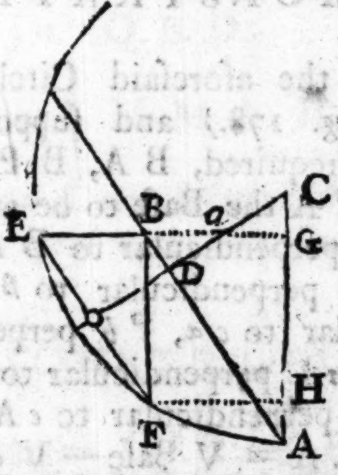
LEMMA I.

The Difference of the verſed Sines of two Arks ×d into $\frac{1}{2}$ Radius or R : = Sine or S *$\frac{1}{2}$ Sum of the Arks multiply'd by the Sine of $\frac{1}{2}$ Difference of the Arks.*

DEMONSTRATION.

Let A F, A E be the 2 Arks ; the Difference of the verſed Sines is A G — A H = F B. And the Sine of $\frac{1}{2}$ Difference of the Arks, *viz.* Sine of $\frac{1}{2}$ F E = F O; and Sine of $\frac{1}{2}$ Sum or Z of the Arks = A D (E P being = F A.) But the Triangle A C D is ſimilar to the Triangle E F B (For the △ *a c* G is ſimilar to the △ E *a o*,)



therefore $AC : AD :: EF : FB$: and $AC \times FB = EF \times AD$, and dividing by 2, $\frac{1}{2} AC \times FB = FQ \times AD$. Q. E. D.

AXIOM IV.

As Rectangle, or Product of the Sines of the Legs. : Square of the Radius : : Difference of the verſed Sines of the Baſe, and of the Difference of the Legs. : Verſed Sine of the Vertical Angle.

DEMONSTRATION.

Reſume the aforeſaid Circles of Paſtboard, (*Pag.* 178.) and ſuppoſe there B the Angle required, B A, B P (= B M) its Legs, P A the Baſe to be any Way oblique, not perpendicular to B A as before. Let fall $P\beta$ perpendicular to βm, and πy perpendicular to $c\alpha$, Pb perpendicular to bA, and mL perpendicular to cA; then will βb be perpendicular to cA; and $bL = bA - LA = V$ Baſe $- V$ difference of the Legs. The Triangles LDm and βbD are ſimilar, therefore $LD : Db :: Dm : D\beta$, and by compoſition $LD + Db = Lb : LD :: Dm + D\beta = \beta m : mD$. But LDm is ſimilar to AEC, therefore $AE : AC :: LD : Dm :: Lb : \beta m$. Alſo the Triangle PBF is ſimilar to the Triangle $\pi y C$ (the Angles $P \beta F$ and $\pi y C$ being right, and the Angles at F and C = from the Inclination of the Plains) therefore $PF : F\beta :: \pi C : Cy$, and by Diviſion $PF - F\beta = \beta m : PF$ or $Fm :: \pi C - Cy = y\alpha : \pi C$, or $y\alpha$; and multiplying the correſponding Terms of both Proportions, *viz.*

$$\left.\begin{array}{l} AE : AC :: bL : \beta m \\ m F : \alpha C :: \beta m : y\alpha \end{array}\right\} \text{There will ariſe}$$

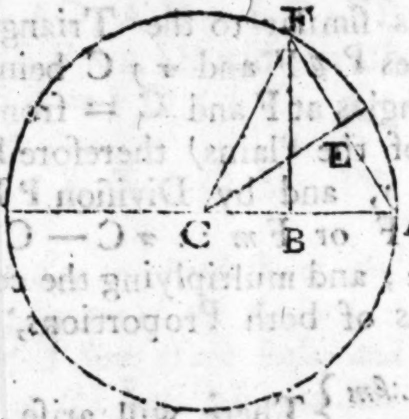
$AE \times mF : AC \times \alpha C :: bL \times \beta m : \beta m \times y\alpha :: bL : y\alpha$. Q. E. D.

LEMMA II.

The verſed Sine of any Ark multiply'd into $\frac{1}{2}$ the Radius, is equal to the Sine ſquare of $\frac{1}{2}$ the Ark.

DEMONSTRATION.

The Triangles CAE and FBA being ſimilar, $BA : AE :: FA : AC :: \frac{1}{2} FA : \frac{1}{2} AC :: AE : \frac{1}{2} AC$ *ergo* $BA \times \frac{1}{2} AC = AEq$,



$= AEq$, *i. e.* the verſed Sine $\times \frac{1}{2}$ Radius $=$ ſquare of the Sine of $\frac{1}{2}$ the Ark. Q. E. D.

CASE XI.

The 3 Sides of any Spherick Triangle being given, to find an Angle.

The Rectangle of the Sines of the Legs *Square of the Radius* : : S, $\frac{1}{2}$ Baſe + $\frac{1}{2}$ Difference of the Legs × S, $\frac{1}{2}$ Baſe — $\frac{1}{2}$ Difference of the *Legs.* Sq $\frac{1}{2}$ *Angle.*

DEMONSTRATION.

By *Axiom* 4, (*See Fig. p.* 178) $AE \times mF : Rq :: bL : ya :: bL \times \frac{1}{2}R : ya \times \frac{1}{2}R$. But $bL \times \frac{1}{2}R =$ S $\frac{1}{2}$ Baſe + $\frac{1}{2}$ Difference of the Legs × S $\frac{1}{2}$ Baſe — $\frac{1}{2}$ Difference of the

Legs (by *Lemma* 1.) and $y\,a \times \frac{1}{2} R = S\,q$: $\frac{1}{2}$ Angle (by *Lemma* 2.) therefore $AE \times mF : Rq :: S$, $\frac{1}{2}$ Baſe + $\frac{1}{2}$ Diff. Legs × S $\frac{1}{2}$ Baſe — $\frac{1}{2}$ Difference Legs. $S\,q\,\frac{1}{2}$ Angle Q. E. D.

CASE XII.

The Three Angles being given to find a Side.

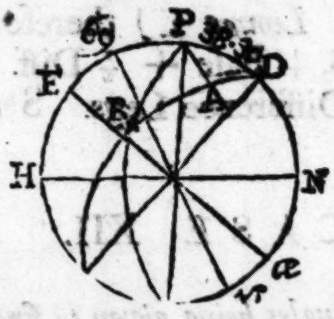
The Angles adjacent to the Side required, call'd Legs, and the Angle oppoſite call the Baſe, then Work as in the laſt Caſe; for ſuch is the Operation in the ſupplemental Triangle, whoſe Angles and Sides are equal to the Supplements of the Sides and Angles of the Triangle propoſed; but Arks and their Supplements have the ſame Sines and Tangents.

Aſtronomical Problems;

Being the moſt uſeful Caſes in Oblique angled Triangles.

CASE I.

In the Oblique angled Triangle B P D, *given the Sun's Co-altitude* B P = 46° 11′ *the Co-latitude* P D = 38°. 30′. *and* D *the Time from*



from Noon. Requir'd **P B D** *the Angle of the Sun's Poſition, or that Angle which the Sun's Azimuth makes with his Circle of Declination.*

S, B P : S, D : : S, P D : S, B

S, B P = *9. 858272*

S, D = *9. 698970*
S, P D = *9. 794150*

Sum = 19. 493120
S, B = *9.634848* **= 25°, 33′ or 154°.**
(27′. Ambi.

CASE

CASE II.

In the ſame Triangle, Given PD, *and the Angles* D *and* B: *Requir'd* B P *the Sun's Co-Altitude.*

S, B : S, D P : : S, D : S, B P

S, B = 9. 634848

S, D P = 9. 794150
S, D = 9. 698970

Sum = 19. 493120
S, B P = 9. 858272 = 46°. 11′.

CASE III.

In the Triangle B P D, *given* P D *the Complement of the Pole's Elevation* = 38°. 30′. B P *the Sun's Co-Altitude* = 46°. 11′. *and the Angle* B = 25°. 33′. *Requir'd* B D *the Complement of the Sun's Declination.* (*See Fig.* p. 202.)

By *Caſe* 4. R : Σ, B : : T, B P : T, B A
By *Rule* 2. Σ, B P : Σ, B A : : Σ, D P : Σ, D A

Draw the Perpendicular A P (by *Prob. 6.)* from P the Extream of the given Side B P and oppoſite to the given Angle at B, which will reſolve the Oblique angled Triangle into two Right ones, which are ſolv'd by the two foregoing Proportions.

R = 10.

Σ, B = 9. 955307
T, B P = 10. 017944

T, B A = 9. 973251 = 43°. 14′.

Σ, B P = 9. 840328

Σ, B A = 9. 858635
Σ, P D = 9. 893544

Sum = 19. 752179

Σ D A = 9. 911851 = 35°. 17′.

Then here becauſe P A falls within the Triangle, B A + A D, *i. e.* 43°. 14′ + 35°. 17′ = 78°. 31 = B D. required.

CASE

CASE IV.

In the Oblique angled Triangle ZPS; *Given the Right-Aſcention, Declination and Altitude of a Star, ſuppoſe at* S: *Required it's Azimuth,* viz. *Let the Star be* Aldebaron, *whoſe Right Aſcenſion* CR = 64°. 17′. *Declination* RS = 15°. 48′. *Co-Elevation of the Pole* = 38°. 30′. *as alſo his Altitude* AS = 30°. *are all given: Required the Angle at* Z? *This is ſolv'd by* Caſe II.

$$\text{Diff. } cr. = 21^\circ.\ 30'$$
$$\tfrac{1}{2}\,SP = 37^\circ.\ 6'$$
$$\tfrac{1}{2}\,\text{Diff. } cr = 10^\circ.\ 45'$$
$$\tfrac{1}{2}SP + \tfrac{1}{2}\,\text{Diff. } cr = 47^\circ.\ 51'$$
$$\tfrac{1}{2}SP - \tfrac{1}{2}\,\text{Diff. } cr = 26^\circ.\ 21'$$

(*See Fig. Page* 188)

$$S,ZP = 9.794150 = 38^\circ.\ 30'$$
$$S,ZS = 9.937531 = 60^\circ.$$

$$\text{II } cr. = 19.731681$$

$$Rq = 20.$$
$$S,\tfrac{1}{2}SP + \text{Diff.} \times \tfrac{1}{2}SP - \text{Diff. } cr. = 19.517286$$

$$Sq\,\tfrac{1}{2}\,\angle Z = 19.785605$$
$$S,\tfrac{1}{2}\,Z = 9.892802 = 51^\circ.\ 23'$$

Therefore Z = 102°. 46′ that is his Azimuth, is nearly E by S.

Here follows the two Uſeful Caſes, One for the Hour of the Day, the other for the Sun's Azimuth; Done after Sir Jonas Moore's *Method, which is very ſhort and ready.*

I. For the Hour.

Given { BP=46°.11′=☉s *Co-alt.* { *Requir'd* the
PD=38°.30′=*Co-lat.* ∠ BDP =
BD=78°.31′=☉s*Co-dec.* *hour fr. noon.*

38°.30′.0.2058543 } *Compl. Arith.*
78. 31 0.0087015 }
43°.6′. 9.8345948 = ½ *Sum of the Legs of the (Angle.*
3°.5′. 8.7306882 = ½ *Diff. of the Legs.*

18.7798388 = *Sum of all* 4 *Numb.*

9.3899199 = *Sine of* 14°. 12′.
(*which doubled is* 28°. 24

(*See Fig. Pag.* 202.)

II. For

II. For the Azimuth

Given all Things as before, Requir'd the Angle BPD = *Sun's Azimuth from the North.*

38°.30′.0.2058543 } { *Compl. Arith. of the*
46. 11. 0.1417281 } { *Legs.*
42°.20′.9.8283006 = $\frac{1}{2}$ *Sum of the Legs.*
23°.10′.8.8251299 = $\frac{1}{2}$ *Diff. of the Legs.*
19.0010129 = *Sum of all; whoſe*

Half 9.5005064 = *Coſine of* 18°. 27′ = *Sine of* 71. 33. *which doubled gives* 143°. 6′. *theSun's Azimuth from the North of the Meridian at that Time.*

FINIS.

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